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ABILITIES RELATED TO MATHEMATICS ACHIEVEMENT IN GRADE THREE PUPILS

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Abilities Related to Mathematics Achievement in Grade Three Pupils" submitted by Bettina Mary Blackall in partial fulfilment of the requirements for the degree of Master of Education.

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ABSTRACT

Concern for the child who is experiencing lack of success in school mathematics prompted this study of abilities considered relevant to mathematics learning. One of the purposes was to assess within one group of children conservation of area and interior volume, perceptual efficiency in discrimination, memory and representation tasks through the visual, auditory and tactual modalities, and estimation. The second purpose was to determine the relationship of these sets of abilities to mathematics achievement.

Ninety Grade three pupils from four different schools were administered a battery of tests to measure mathematics achievement, conservation, perceptual efficiency, estimation, intelligence and reading.

It was found that the high mathematics achievement group was more successful than the low group on every test. The groups were significantly different on auditory discrimination, memory and closure, as well as on conservation of interior volume, visual discrimination and estimation. When differences in intelligence were accounted for, the groups remained significantly different for the auditory abilities, along with visual discrimination and representation.

Significant correlations occurred between mathematics achievement and eight of the other twelve variables categorized under conservation, perceptual efficiency and estimation.

One major finding of the study was the prominence of the auditory abilities. They were amongst the main predictors of mathematics

achievement arising from a stepwise regression analysis, and appeared in a number of significant correlations with the other variables.

Qualitative and quantitative differences were evident among the children. Although the high mathematics group achieved a higher mean than the low mathematics group on each test, those children categorized as top and bottom performers were distributed over the three mathematics groups. Furthermore, eighty-four of the ninety children performed better than the mean of the sample on at least two of the abilities tested. This is another major finding.

The study concluded with several implications for education and some suggestions for further research.

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I. INTRODUCTION AND STATEMENT OF THE PROBLEM

> Learning ability is by no means a unitary trait ... Different patterns contribute substantially to determining which subgrouping will learn most effectively under conditions of different instruction and task demand... Differences in learning achievement whether measured by intelligence tests or by school achievement, in human beings represent the products of different degrees of goodness of fit between the learner, the task, and, in particular, the instructional mode (Birch, 1968, p. 56).

Many contemporary educators recognize the desirability of finding the best match of particular children in specific circumstances with appropriate learning situations. The effort seems to be just as great in mathematics as it is in any other field of education. In order to achieve this basic goal, more information must be gathered especially about those children for whom conventional mathematics programs seem least appropriate.

The differing abilities which children possess may affect the way in which their learning takes place. Neglect of the particular abilities of some children may have contributed to their lack of success in learning mathematics. Abilities related to mathematical learning formed the broad basis for this research. Conservation, perceptual efficiency and estimation were the three sets of abilities selected for investigation. Spatial concepts provided the unifying theme in the tasks chosen to measure these abilities.

Jean Piaget (1952) contends that "conservation is a necessary

condition for all mathematical understanding (p. 4)." In the past decade and a half, many research projects concerned with Piagetian conservation have gathered sufficient evidence to indicate that the ability to conserve, which generally becomes manifest around the age range of seven to eight years, is related to mathematics achievement. Riggs (1970), Cathcart (1969), Reimer (1968) and Almy (1966) have shown the significant contribution of numerical and measurement concepts with emphasis on length. The conservation of space, both two-and three-dimensional, also needs further investigation in this regard.

Interwoven with the acquisition of conservation is the influence of perception. Perceptual appearances can tend to dominate so that the conceptual invariance of an object under transformation is not recognized. Perceptual functioning, in turn, is closely connected to the modes of instruction common in schools. The emphasis placed on the visual and auditory modalities in teaching situations assumes that all pupils are equally proficient in these modalities. Attention needs to be focused on those individuals who do not follow the common trends reported by researchers. The spatial motif provides the third possible connection between perceptual functioning and mathematics learning.

For these reasons, children's perceptual efficiency across the visual, auditory and tactual modalities needs exploring.

An ability related to both conservation and perceptual efficiency is that of estimation. Listed as an objective in some mathe-

matics curricula, it has usually been translated by textbook writers into a series of abstract exercises involving computation. Yet this interpretation seems incompatible with any mathematics curriculum influenced by the Piagetian emphasis on personal activity and involvement (Duckworth, 1964). Educational research has tended to neglect investigation of the relationship of estimation, whether abstract or experiential, to mathematics learning. On the other hand, psychologists have long studied notions of probability and judgment, both of which may be involved in certain estimation tasks. However such theoretical research is considered peripheral to the kind of estimation activities relevant to young children within the context of an experientially-based mathematics program.

A spatial motif is used as the recurring theme throughout this research project. Spatial abilities which are theoretically independent of verbal abilities (Cattell, 1971) do not depend on a prior understanding of symbols and are readily accessible through experiences in everyday life (Beard, 1964). The dominance of the verbal, whether spoken or written, as the medium of instruction assumes that all children are strong in verbal abilities. On the other hand it is possible that a spatially-oriented mathematics curriculum may furnish a viable alternative for some children. However, the initial task is to acquire information about children's spatial abilities.

The present study was designed to investigate three sets of abilities related to mathematics learning in children, namely, con-

servation, perceptual efficiency and estimation, which are unified by a spatial theme.

II. PURPOSE OF THE STUDY

The purposes of this study were:

1. to investigate within the one group of children
    - a. conservation of area and interior volume;
    - b. perceptual efficiency in the visual, auditory and tactual modalities, specifically with relation to their functions in discrimination, memory and representation tasks;
    - c. estimation of length, area and numerical quantity;
2. to determine the relationship of these domains to mathematics achievement.

III. MAJOR HYPOTHESES

The two purposes stated above gave rise to the following hypotheses.

<u>Hypothesis 1</u>

There is no significant difference between the scores obtained by high, middle and low achievers in mathematics on tests of

i. conservation

ii. perceptual efficiency

iii. estimation.

<u>Hypothesis 2</u>

There is no significant correlation between scores obtained

on tests of conservation, perceptual efficiency, estimation and mathematics achievement.

IV. DEFINITIONS

Mathematics achievement. The subject's score on the mathematics sections of the _Canadian Test of Basic Skills_.

Intelligence. The subject's score on the _California Short-Form Test of Mental Maturity, Form IH_.

Reading Ability. The subject's score on the _Gates-MacGinitie Reading Tests, Form C_.

Conservation. A specific property of an object is said to be conserved if that property remains constant despite transformation in the shape and arrangement of that object.

Transformation. The operation of changing the configuration of an object without changing the specific physical property or properties being studied.

Conserver. A subject who demonstrates understanding of conservation of a specific property by completing the appropriate task or tasks successfully.

Partial Conserver. A subject who demonstrates some understanding of conservation of a specific property, but does not complete the appropriate task or tasks successfully.

Non-conserver. A subject who fails to demonstrate any understanding of conservation of a specific property.

Perceptual Efficiency. The subject's scores on the specific tasks of discrimination, memory and representation in the visual, auditory and tactual modalities.

Modality. Sensory input channel.

Estimation. Ability to determine subjectively a numerical relationship between two quantities without measuring or counting.

Interior Volume. The amount of space or room taken up by an object.

V. LIMITATIONS

In interpreting the data of this study the following limitations should be borne in mind.

i. The sample for the study was selected from a population of Grade 3 pupils in the Edmonton Separate School System, Alberta. It was assumed that this sample was representative of urban children of that grade level.

ii. No effort was made to control for the effects of previous learning which may have affected how a child performed on the various tasks.

iii. In the conservation tests the wording of the questions may have varied slightly in reaction to a subject's responses. This was controlled as much as possible.

iv. Scores on the mathematics sections of the Canadian Test of Basic Skills are assumed to reflect a subject's level of achievement in mathematics.

VI. SIGNIFICANCE

It has been tentatively suggested in the introduction that it may be possible to provide viable alternatives to the mathematics curriculum and instruction presently found in schools, for those children whose capabilities are not matched to their mathematics performance. Because the instructional programs are generally dominated by the verbal, a possible alternative is a spatially-oriented approach. Therefore it seems reasonable to utilize a spatial setting in order to explore abilities which may contribute to mathematics achievement.

The relevance to mathematics learning of the three domains of conservation, perceptual efficiency and estimation has already been indicated briefly and relies quite heavily on Piagetian concepts of child development for justification.

Young (1969) has pointed out that training studies which have concentrated on one factor, such as the conservation of number, have been unsuccessful generally. If alternatives to current programs are to be produced, knowledge of children's abilities on a multi-variate basis will be a necessary condition.

While this study does not supply this lacking information, it may serve as one of the steps in that direction, and lay the basis for the future development of a mathematics curriculum more appropriate to the needs and learning styles of some individual pupils.

VII. THE EXPERIMENTAL SETTING

The following is an overview of the experimental design. A more detailed account is reported in Chapter III.

The population from which the sample was drawn consisted of the Grade 3 pupils in four elementary schools of the Edmonton Separate School Board. The sample was made up of ninety children selected from this population on the basis of mathematics achievement.

Instruments to test the abilities listed below were administered to each subject, either in group situations or individually:

i. conservation of area and interior volume;
ii. discrimination, memory and representation in the visual, auditory and tactual modalities;
iii. estimation.

As well, mathematics, intelligence and reading scores were obtained for each subject.

The data were analyzed for correlations along with one-way analysis of variance and two-way analysis of covariance, stepwise regression and factor analysis.

VIII. OUTLINE OF THE REPORT

A review of the relevant literature will be presented in Chapter II. Chapter III contains a detailed account of the experimental design, testing procedures and the research procedures used

to test the hypotheses. The results of the data analysis are contained in Chapter IV. The final chapter, Chapter V, includes a summary and discussion of the findings, and contains some implications for education and future research.

CHAPTER II

REVIEW OF RELATED LITERATURE

Conservation, perceptual efficiency and estimation have been proposed as skills related to a child's mathematical development. The purpose of this chapter is to review the studies on the acquisition of conservation of area and interior volume, a feature of the transitional stage between Piaget's pre-operational and concrete operational thought. Investigations examining perceptual efficiencies, organized on a three-by-three matrix of modality and phase, are summarized and discussed. Mention is made of the limited research carried out on estimation, as defined in this study. The relationships between these variables and mathematical learning are dealt with incidentally.

I. CONSERVATION OF AREA AND INTERIOR VOLUME

Piaget's developmental theory has been summarized by Flavell (1963), and in numerous studies, for example, Cathcart (1969), Reimer (1968) and Towler (1967). The section of the structure relevant to this study is the dynamic and fluctuating transition from the stage of pre-operational thought to that of concrete-operational thought. A feature of this period is the child's gradual acquisition of the principle of conservation pertaining to quantity, number, substance, length, weight and area, designated as "first-order" conservations because they require only single applications of

reversibility.

The child in the pre-operational stage tends to be dominated by his perceptions and is often unaware when his conclusions are self-contradictory. The child who has attained conservation can see beyond the immediate appearance of things. His thought becomes more conceptual. He can cancel out the effects of change in order to focus on the elements of an object that have remained unchanged (Almy, 1966). He can integrate two temporally separate experiences into a single judgment (Reimer, 1968). These conservations therefore can be considered indexes of concrete-operational thought (Brainerd and Allen, 1971).

In between these two stable states, a child vascillates in his grasp of the notion of invariance. He may be confident of the numerical permanence of a group of beads, yet change his mind with astounding regularity when faced with transformations of a set of building blocks.

The extent of this transitional period can be inferred only indirectly from research reports. The one study claiming to be longitudinal was limited to kindergarten children who were re-interviewed about quantity and number at six-monthly intervals for two years (Almy, 1966). No research seems to have been directed towards investigating just how long a particular individual is operating in this state of inconsistency. Yet this information should be important to curriculum designers.

Sawada (1966) has shown that some children can conserve length as early as four years of age. At the other end of the scale,

eight per cent of eleven-year olds were unsuccessful on the test for conservation of substance (Elkind, 1961).

In the main, research investigating the presence or induction of first-order conservation has concentrated on number, length, substance and weight. Very little has been done on area. In fact, this aspect of conservation is not categorized in the "Interpretive Study of Research and Development in Elementary Mathematics" (Suydam and Riedesel, 1969), nor in the preceding dissertation "An Evaluation of Journal-published Research Reports on Elementary School Mathematics 1900-1965" (Suydam, 1967).

a. Conservation of Area

Piaget et al. (1960) devised some ingenious problems to investigate the conservation of area. In the Barns Test, two pieces of card demonstrably equal in area represent two fields, in each of which a cow is pasturing. Barns of equal dimensions are, in one field scattered, and in the other clustered near an edge. Through this situation Piaget probes a child's understanding of the invariance of area upon subtraction (p. 262).

In another problem, a segment is removed from a rectangular piece of card and attached at a different place. The child must ignore the increase in perimeter in order to acknowledge the equality of area (p. 273).

In a variation of this type of transformation, two congruent triangular pieces made from a rectangular piece of card are rearranged into a large triangle and other shapes such as a parallel-

ogram in order to check a child's ability to withstand the apparent change in size (p. 274).

Beard (1960) paralleled the Transformed Triangles Test by the use of a form board, in which rectangular and rhombic holes could be exactly filled by the same two congruent triangular wooden blocks. Six- and seven- year old children were asked which hole they thought was bigger and then saw the blocks fitted in. The rather vague reporting indicates that of the sixty Ss, twenty-three persisted in their perceptual judgment that the rhombic hole was larger.

In a comparative study of Ghanian and English children, Beard (1965) cut two congruent squares in half, one parallel with a side, the other diagonally. A triangular half and a rectangular half were then combined to make a "house", which was compared with the size of the original square. At age eight, both groups had a success rate of thirty-two per cent. At age ten, fifty-six per cent of the Ghanians were successful as against sixty-two per cent of the English children.

In their study aimed at the training of Grade 1 and Grade 3 pupils in the measurement of length and area, Beilin and Franklin (1962) used the Barns Test as well as the Rearranged Rectangles Test but failed to report the specific data. In two further experiments, Beilin (1964, 1966) used an analogue of the Rearranged Rectangles Test in the form of a Visual Pattern Board. By the insertion of prepatterned templates, transformations of connected or unconnected unit squares could be instantly presented, without any motion being

evident. No physical manipulation was involved. Apart from their perceptual impressions, Ss could use counting (iteration) or translocation in making their judgments of equality of area. Beilin's discussions concentrated on the logical and infralogical operations which need to intervene actively in the process of perception. In the 1964 study, fifty per cent of the fourth graders (N=26) could not judge correctly area equality. This percentage was reduced to forty in a later review of the experiment (Beilin, 1966). The 1966 experiment was restricted to first and second grade children. The number of area conservers on the pre-test (N=39: Grade 1 = 11, Grade 2 = 28) was raised after training to seventy-seven (Grade 1 = 27, Grade 2 = 50) out of a total of two hundred and thirty six children, equally divided between the two grades.

Murray (1965), who describes conservation as "a process of resisting illusion or of recognizing a disparity between appearance and reality" (p. 62), presented Grades 1, 2 and 3 children with illusion-distorted areas. Seventeen of the twenty-three Grade 3 children were successful on the Delboeur concentric circles illusion. He claims the data support the conclusion that the transition from non-conservation to conservation occurs between the ages of seven and eight. No physical manipulation of the problems was allowed by the Ss.

Kempler (1971) proposed that categorization of rectangles as large or small is a valid alternative to verbal enquiry for differentiating those children who judge regions as two-dimensional

entities from those who essentially make "tall-short" judgments. Except for the Grade 5s whose judgments, Kempler suggests, are affected by their having recently learned the rule for computing area, the results show a steady increase in accuracy with grade level.

Several researchers have investigated the conservation of area incidentally. Goldschmid (1967) used a variation of the Rearranged Rectangles Test on Grades 1 and 2 normal and emotionally disturbed children. His results indicate that area is easier to conserve than length. Cathcart (1969) who used the Transformed Triangles Test with Grade 2 and Grade 3 children arrived at the same conclusion (p. 85). Goodnew and Bethon (1966) found on an adapted Barns Test that average eight-year-olds scored a .38 proportion of correct answers compared with .56 for average eleven-year-olds. Superior children of the same ages achieved .65 and .94 respectively. Verrizo (1970) was also interested in the effect of intelligence. Of one hundred and forty gifted children in Grades 3 to 8, only one child (in Grade 8) was not a conserver of area, though sixteen pupils in the lower grades could not handle complementary area. Unfortunately the actual test situations are not described.

In a scalogram analysis of Piagetian area concepts, Needleman (1970) reports that the total sample of sixty-nine boys in Grades 3 to 8 attained the concept of conservation of area. The Ss ranged in age from 8.6 to 14.4 years with I.Q. scores from 100 to 120. Even the superior eight year olds in Goodnow and Bethon's (1966) study had a one-third failure rate.

The paucity of research on the conservation of area is obvious. Yet one conclusion important to curriculum designers can be drawn. Although a great number of children may not achieve this ability before the age of ten or eleven years, there are many younger children, not necessarily superior in intelligence, who are confident of the invariance of area under transformation.

b. Conservation of Interior Volume

A similar paucity is evident in the research on the conservation of interior volume, which Piaget et. al. (1960) define as "the amount of matter contained within the boundary surfaces" of an object (p. 360). Interior volume is distinguished from occupied volume which is the amount of space occupied by the object as a whole in relation to other objects round about (p. 360), when, for instance, an object is immersed in a container of water. Complementary to occupied volume is displacement volume which refers to the amount of water within the container (p. 358). The more sophisticated understanding of occupied and displacement volume is associated with the advent of formal operational thought, whereas interior volume is categorized with length and area (p. 355).

In the classic Piagetian test for interior volume, various constructions are rebuilt from an original model of thirty-six unit cubes, either by the child or by the experimenter. The storyline tells of lakedwellers whose houses are rebuilt on different sized islands after a storm. Equality of volume is expressed by the words

"as much room." Success was reported for two children aged 7.7 years, (p. 356-357).

Many researchers for example, Beard (1965), Uzgiris (1964), Sigel, Roeper and Hooper (1966), Cathcart (1969), Verizzo (1970), have tended to disregard the dichotomy within the property of volume and leave the reader to clarify which aspect is being studied. Generally it turns out to be displacement volume with the use of containers of water, into which objects are dropped. Other researchers seem to be confused. An example is Elkind (1961) who questioned children on the amount of room taken up by the "hotdog" that was once a ball of plasticine. The plasticine, no matter what its shape, is not seen "in relation to neighbouring objects in space" (Piaget, et al., 1960, p. 375). The task description indicates he is dealing with interior volume. No object other than the plasticine is introduced, yet he seems to compare his findings with Piaget's statements on occupied and displacement volume. Uzgiris (1964) makes similar comparisons.

On the other hand the distinction between interior volume and substance is not so clear. Piaget et al. (1960) state, "Looking at it from the physical point of view, we may say that the conservation (of interior volume) is based on the invariance of a quantity of matter" (p. 370). Qualitative operations underlie the topological invariance of content or interior volume (p. 370). For Elkind (1961) the criterion lies in a question: "Do they both have the same amount of 'clay'?" (substance) or "Do they both take up as much room?"

(volume). The difference is evident from the responses. Ninety-two per cent of the eleven-year-olds were conservers of substance but only twenty-five per cent managed the invariance of interior volume.

Lunzer (1960) replicated the Islands Test as closely as possible in order to verify the general correctness of Piaget's findings. Of the four children tested at each age, one only was successful at age seven, two at age eight, while the ten year olds and on were all successful.

Lovell and Ogilvie (1961) repeated the experiment with twelve unit cubes. Conservation following a reconstruction ranged from sixty-five per cent of the first year juniors (approximately eight years of age) to ninety-three per cent of the fourth year pupils. Goldschmid (1967) replicated this version which he labelled a test of three-dimensional space. He reports that it was the most difficult of the first-order conservations, except for that of distance.

At odds with Piaget's sequence in the acquisition of conservation of volume is Lister's (1970) finding that seven of her one hundred and four educably subnormal Ss gave "clear conservation answers in the displacement volume situations but non-conservation answers for interior volume." (p. 58) However she admits her displacement volume situations were "slightly different" from those described by Piaget et al. (1960), in that the objects were not immersed in the water at the time of the transformation.

Only a small number of children have participated in the research reviewed. Their performance on conservation tasks do not

contradict Piaget's assertion that interior volume is conserved within the same developmental period as length and area. There are not enough data to suggest a wide range in the ages of those acquiring understanding of this invarance of interior volume.

c. Related Studies

Many training studies have been carried out, but Beilin's (1966) is the only one reported on the conservation of either area or interior volume (Brainerd and Allen, 1971).

Cathcart (1969) summarized the role of conservation in mathematics achievement. Dodwell (1961), Hood (1962), Almy (1966), Steffe (1966) and Reimer (1968) all found a significant and positive relationship. Of these studies only Almy's (1966) extended the age range as far as Grade 2 and the properties involved were mainly number with some quantity and length. When the differences in intelligence were accounted for Steffe (1966) found a positive relationship between conservers of substance and mathematics achievers at a Grade 4 level.

Cathcart (1969) points out that non-conservers can memorize facts as well as conservers and therefore the type of achievement test used is of crucial importance (p. 45).

In general the literature lends support to Piaget's (1952) contention that "conservation is a necessary condition for all rational activity" (p. 3) and for mathematics in particular.

> A continuous quantity such as a length or volume can only be used in reasoning if it is a permanent whole, irrespective of the possible arrangements of its parts. In a word, whether it be a matter of continuous or

> discontinuous qualities, of quantitative relation perceived in the sensible universe, or of sets and numbers conceived by thought, whether it be a matter of the child's earliest contacts with number or of the most refined axiomatizations of any intuitive system, in each and every case the conservation of something is postulated as a necessary condition for any mathematical understanding (p. 3).

II. PERCEPTUAL EFFICIENCIES

Perception is often seen as the influence which is holding back the conceptual development of the child. To quote Hyde (1970), Piaget's theme is "the gradual liberation of the child's thinking from perceptual evidence (p. 40)." Piaget himself declares "perception even at its best (is) forever a probalistic approximate affair whose products are constantly at the mercy of changes in the field conditions"(Flavell, 1963, p. 233). Vernon (1962) acknowledges that the visual pattern which impinges on the brain is not static -- it continually moves and flickers. Yet, she points out, the essential feature of the world as we perceive it is its constancy and stability (p. 14). Piaget himself agreed that in perception it is the constancy phenomena which are the closest to the phenomena of conservation (cited Hyde, 1970, p. 39). For Zaporozhets (1965) "perceptive images gradually reflect the environments more fully and adequately" (p. 647).

Elkind (1967) expresses the relationship of Piagetian conservation and perception at the same time reconciling these views to those of the English and Russian authorities.

> The essential characteristic of the development is a gradual shift from perception that is controlled by peripheral sensory processes to perception where central processes play the leading role (p. 358).

The study of perception has produced a variety of theories which have been rigorously analysed and vigorously defended by psychologists such as Gibson and Gibson (1955), Bruner (1957), Wohlwill (1960), Gibson (1969). Along with the Russian psychologists (Zaporozhets, 1965), these Americans reject the static receptory concept because "somehow we get more information about the environment than can be transmitted through the receptor system" (Gibson and Gibson, 1955, p. 33). "At all levels, perception is active and cannot be reduced to passive reception" (Piaget, 1968, p. 141). It is a process with a screening or gating component which selects or filters sensory input from the continuous bombardment of external stimuli. A discrepancy exists between the sensory input and the finished percept. The Gibsons (1955) concluded that the process must be one of supplementing or interpreting or organizing. Wohlwill (1960) proposed that it is "a decision process partaking to a greater or lesser extent of the characteristic of an inference from the sensory information given." (p. 252) For Bruner (1957) it is "an act of categorizing" (p. 123).

Fellowes (1968) supplies a basic definition which would be accepted, perhaps grudgingly, by the psychologists mentioned. Perception is "the process by means of which an organism receives and analyzes sensory information" (p. 3).

"From the five senses, formal education in the main, utilizes

only three, namely the auditory, visual and kinaesthetic" (Saunders, 1931; cited Eagan, 1970, p. 14). Glusker (1969-70), a neuropsychologist, designed a theoretical model relating the various aspects of language learning. His model presents these three perceptual modes, each of which has three phases -- discrimination, memory and integration (p. 119-120). Eagan (1970) points out that this model can be extended to reading (p. 14). That it can be extended to include mathematics learning has not yet been investigated or reported. In their study of children failing the elementary grades, Sabatino & Hayden (1970) incidentally attempted to relate various perceptual efficiencies to mathematics achievement which was included as one of the variables.

The three phases are not isolated from other aspects of the human person, nor are they so clearly differentiated from each other, as shall be seen later when memory is discussed.

a. Discrimination

The act of discrimination presupposes a minimum level of acuity, unless children with low thresholds are especially catered for as in Eagan's (1970) study. It also assumes that the child has oriented his receptors towards the stimuli, as the result of a scanning activity (Fellowes, 1968, p. 4) which carries out a gross sorting process, prior to concentrating on the "distinctive features (which) can be thought of as dimensions of difference which distinguish and provide contrasts among subjects" (Pick, 1965, p. 332). Moreover

discrimination, along with memory and representation, cannot be judged to have occurred unless there is some overt response mostly expressed in a different mode.

Most of the research on visual discrimination has concerned letter-like forms (Gibson et al., 1962; Pick, 1965), conventional print (Gibson, 1965, Bonsall and Dornbush, 1969), as well as geometric shapes (Babska, 1965) and pictures of objects (Boguslavskaya, cited by Zaporozhets, 1965). There appears to be agreement that after a rapid surge at about three years of age, children improve gradually in their accuracy till errors drop to near zero at age seven to eight years, thus indicating a developmental trend. Of the studies mentioned, only Bonsall and Dornbush (1969) reports having taken intelligence into account.

Somewhat different from the discrimination tasks which call for comparisons are the tests of imbedded figures which require the search for parts within wholes (Ghent, 1956; Satterley, 1968; Elkin et al., 1964, 1970). Similar results are obtained, except for psychoneurologically impaired children.

A great many studies have shown a strong relationship between auditory discrimination and reading achievement, as well as intelligence up to the age of eight to nine (see summaries of research in Bruininks, 1968; Eagan, 1970; Wepman, 1970; Vernon, 1970). In her longitudinal study of auditory discrimination, Thompson (1963) noted that twenty-four per cent of the sample had inaccurate auditory discrimination at the end of the second grade. The tests included

Wepman's Test for Auditory Discrimination.

The connection between auditory discrimination and mathematics achievement may rest on the fact that most instruction, especially in the early years, is in that mode (Saunders, 1931, cited Eagan, 1970, p. 14).

In tactual discrimination, the sense organ or receptor is the hand, whose movements are exploratory rather than performatory, thus providing a definite channel of information about the external environment (Gibson, 1962). Piaget and Inhelder (1967), of which the first English language edition was published in 1956, observed the tactual or haptic exploration of objects and shapes by children set an identification task. The children were asked to select the same object or shape from a collection presented visually (p. 17-22). By the age of three and a half years familiar objects followed by topological shapes were more or less easily identified, but not euclidean figures, which were progressively differentiated between four and a half and six or seven years. Only at the age six and a half to seven years was the synthesis of complex forms achieved (p. 20).

Page (1959) followed the Genevan experiments as closely as possible with about sixty children aged 2.10 to 7.9. He confirmed the previously reported findings.

Two studies link tactual discrimination tasks with the learning of mathematics. Gibson (1962) suggested that the spatial properties of solid geometric forms can be registered satisfactorily through active touch. Hoop (1971), an occupational therapist who evaluated the manipulation used during exploration of objects, recommended guided systematic training in order to improve object recognition.

b. Memory

A major problem in the phenomenon of memory is the intuitively recognized discrepancy between the severely limited span of immediate memory and the vast amounts of information man can remember over long periods of time. Chunking or organizing the verbal input into ever increasing schemas and the two factor theory of short and long term memory are two of the explanations being currently pursued (Jackson, 1970, p. 41).

Having explored the literature on memory, Jackson (1970) argued that it is "a process of reconstruction or recreation rather than simply the reproduction of an input event" (p. 17). To reconcile the intuitive and theoretical views of memory, Sinclair (1971) who is affiliated with the Genevan School, postulated two types of memory, both concerned with an object or situation already encountered. The essential difference seems to be the presence or near presence of the event at the moment of recall as against its absence. The former she labels "Recognition Memory." The latter she terms "Evocation Memory" because it is "a kind of representation and consists in the evocation of situations already encountered" (p. 127). Piaget et al. (1968) distinguished further between Evocation Memory, the highest level, and Reconstruction Memory.

> Reconstruction memory, which we can observe when we give the subject the material which was used to construct the model (along with some extra material) and ask him to rearrange the material in the configuration which he had seen earlier (p. 12).

With its strong component of representation and organization, Evocation

Memory seems to be the kind Jackson (1970) was discussing, and near to Glusker's (1969-70) third modal phase of integration. Recognition Memory appears more closely akin to the immediate recall tasks of the Illinois Test of Psycholinguistic Abilities and the Weschler Intelligence Scale for Children.

Memory of this sort was one of the factors in the study by Gibson and Gibson (1955) who used a set of eighteen scribble items which included four copies of a "standard" scribble. The Ss, adults, eight and a half to eleven year old children and younger children of six to eight, were shown the standard item for five seconds then asked to select exact copies from the pack. Whereas the adults made a mean of three mistakes, on the first time through, the older children made a mean of 7.9 mistakes and the youngest group 13.4.

Miller, Bruner & Postman (1954) found that familiarity of letter sequences increased the number of letters recalled after tachistoscope presentation.

Although there are some studies on the relationship of visual memory and reading, there seems to be none directly related to mathematics achievement.

Auditory memory in connection with reading achievement has been a favorite topic with researchers as evidenced by the bibliographies in Gibson (1969), Walby (1967), Eagan (1970), and Vernon (1971). The common finding has been that poor readers are likely to perform unfavorably on auditory memory tests, especially up to the age of eight or nine. Older retarded readers exhibit the same tendency.

Whether there is any relationship with mathematics learning apart from the fact that the auditory mode dominates instruction is yet untested. Knowledge about the relationship of mathematics achievement and tactual memory is in a similar state. The only study found in the literature which involved tactual memory at all investigated preschool children after six and twelve months following a training program. Coyle (1968) found retention had occurred whether the children had been instructed individually or in groups.

Of interest is Luria's statement that children under five years old could remember the differences between pairs of shapes more easily when they handled them, which suggests that visual perception of contour may be reinforced or improved by tactile perception (cited by Vernon, 1971, p. 12).

c. <u>Representation</u>

Observation of the lag between the ability to differentiate visual patterns and the ability to reproduce them has caused psychologists to consider if cognitive factors are involved, rather than mere immature motor ability. Piaget (1968) postulated that the child develops mental images as symbols of reality drawn from his own actions. When asked to copy a shape he is representing not that perceived shape, but the mental image which is affected by his level of development (p. 7)

> The drawing, like the mental image, is not simply an extension of ordinary perception, but is rather the combinations of the movements, anticipations, recon-

> structions, comparisons and so on that accompany perception, and which we have called perceptual activity (Piaget and Inhelder, 1967, p. 33).

Bartlett (1932) maintained that what was remembered and represented was not the event itself but a general rule of arrangement or schema of the event and that this schema was used as a basis for the reconstruction of the original event (p. 213). Accurate recall is the exception rather than the rule since reconstruction of the event is based not simply on the input but on past learning and knowledge.

Olson (1970) concluded that

> ...it should be clear that copying of a visual form or its reconstruction in any other medium reflects not a simple "printout" of a perceptual input, but, rather, a conceptual schemata that has been reconstructed by the child's reflection upon his own activities and by the influence of the culture through the pictorial record of past inventions (p. 19).

Piaget and Inhelder (1967) took up Bartlett's notion of reconstruction when speaking of spatial knowledge. "The children are able to recognize, and especially to represent only those shapes which they can actually reconstruct through their own action" (p. 43).

In order to investigate the development of visual representation, the Genevans prepared a set of twenty-one figures to be copied by drawing. The models were of three kinds. Some emphasized topological relations while others were simple euclidean shapes. Combinations formed the third group (p. 53). For children 3.6 to four years it was only topological relationships which were indicated with any degree of accuracy. Four year olds were beginning to distinguish curved shapes from straightsided ones, but the square and triangle remained

undifferentiated. Most often the oblong was the first reproduced correctly (p. 56). Later these children separated pairs of shapes such as the circle and ellipse. The rhombus was drawn correctly just before the age of 6.6 to seven years (p. 56). Piaget confidently proclaimed that "topological relations universally take priority over euclidean relations" (p. 67).

Piaget and Inhelder used the drawings of the rhombus to demonstrate that to construct a euclidean shape something more than a correct visual impression is required. The relative order of the parts lying each side of a rhombus's central axis involves the reversal in the symmetry of one of the component triangles (p. 75). In other words some form of transformation of mental images needs to occur.

From a replication study by Ferns, Peel (1959) confirmed the Genevan sequence of phases as well as the order of difficulty of the items.

Lovell's (1959) replication study had as Ss one hundred and forty-five children aged 2.11 to 5.8 years. His findings are somewhat confirmatory. Topological figures were easier to draw than euclidean, but not if the latter had curved edges. The variability in performance within an age group was far greater than amongst the Swiss Ss.

Graham, Berman and Ernhart (1960) studied the changes with age in the ability of preschool children to make pencil and paper copies of simple forms presented visually. The stimuli consisted of eighteen forms, including the four "primary" figures -- square, triangle, circle and rhombus -- and various combinations of parts of them.

Children's reproductions were measured for accuracy. Of the straight-edged euclidean shapes, the square was best handled, followed by an equilateral triangle on its base, then an equilateral triangle on its vertex. The rhombus was the most difficult. The children's reproductions showed increasing accuracy with age but the order of difficulty was highly consistent in each of the age groups.

Unfortunately none of these investigators of visual representation took note of intelligence or included any children older than seven years, as if it is assumed that all children beyond this age are achievers on such tasks. Such an assumption needs questioning in the light of contemporary mathematics curricula which include a geometry strand throughout.

In the auditory mode, representation can be judged by the ability of children to respond successfully to closure tasks. The input is a series of sounds, not restricted to phonemes, which the child has to organize mentally into words. According to Bannantyne (1971-2), when a person hears a word, he is in fact receiving a group of sounds which have to be assembled into a single unit.

Hardy (1965) cited Munroe, Bond, Gates, Vernon, Chall, Hildreth, and Tinker, all highly respected researchers in the field of reading, as agreeing that auditory synthesis is a significant factor in reading achievement. Her own study confirmed the relationship, as did Kass's (1962).

As with visual representation, the little research on tactual representation has concentrated on geometric shapes. In their investigation of haptic perception, Piaget and Inhelder (1967) asked the

children to draw the shapes they had explored with their hands. At each stage representation lagged behind discrimination (p. 30). However there was a strong similarity between the drawings of shapes perceived tactually and visually. The ages at which rectilinear shapes were differentiated from curvilinear were the same. The rhombus was satisfactorily drawn by a child of 6.11 years (p. 35).

In replicating this experiment, Page (1959) had children draw shapes explored tactually. He confirms Piaget's findings generally. Although the rhombus was one of the forms, he did not mention the responses of the children to this or any other euclidean figures.

d. Summary

Some conclusions can be drawn from this overview of research on the efficiency of perceptual outcomes. First the field is dominated by investigators interested in the process of reading. For his dissertation, Bruininks (1968) could cite thirty studies concerning the relationship of reading with various perceptual skills. It was only in the period after the publication of Piaget's relevant works in English, that mathematics educators showed any interest in studying the perceptual functioning, and its relationship to mathematics learning.

Secondly, studies seem to concentrate on children up to the age of eight years.

This situation indicates a need for research to explore the possible relationship between mathematics learning and perceptual efficiency. A broader age range amongst the subjects also seems

desirable.

III. ESTIMATION

The ability to make estimates, whether of size, shape or relative frequency, underlies a great deal of everyday behavior. Estimation is listed as an objective in many mathematics curricula. Yet it is usually based, not on physical experience of relationships between quantities as in the present study, but on abstract comparisons using computation. This tendency, which appears to ignore the existence of a stage of concrete-operational thought, is reflected in the little research reported. Suydam (1967) offers two studies, neither of which is classified as satisfactory research. Dickey (1934) found no significant difference between Grade 6 groups who practised estimating answers to arithmetic problems and examples, and those who did not. Both groups gained in achievement. Faulk (1962) was interested in the range of answers which Grade 5 students (N=52) gave in estimating solutions to verbal problems.

Suydam and Riedesel (1969) list no reported studies in the category Estimation, but do cite Nelson's dissertation (1969). Fourth and sixth grade pupils (N=623) were taught procedures for estimating answers to problems or exercises by the process of rounding the numbers. The sixth grade children significantly surpassed the control group in arithmetic competence, though the fourth grade control group was superior in computation. The observation was made that because the experimental pupils took time to estimate they did not attempt or solve as many test items as the controls.

Though the Ss were college-bound Grade 11 students, Paull's (1971) investigation into the ability to estimate in mathematics has some relevance to the present study. The Tests of Estimation covered area, length and numerical computations. The ability to estimate answers to arithmetic problems was positively correlated with mathematical ability and verbal ability. By contrast, the ability to estimate area and length did not correlate with the ability to do mathematical computations rapidly.

Experimental psychologists have long investigated aspects of judgment such as probable cue situations and stimulus components, particularly form and colour. Their findings are only distantly related to our understanding of estimation in the context of elementary mathematics influenced by Piagetian research. Nor is estimation "a decision-making process in uncertain situations where the individual must formulate and operate upon subjective estimates of likelihood" (Hecox and Hagen, 1971, p. 107). Neither is it subject to the emotional value of the stimulus (Tajfel and Winter, 1963). Therefore research in any of these psychological aspects is peripheral to the present study.

Estimation of length is a cognitive task which includes some measurement concepts (Tayal, 1972). Given a reference unit as a 'part', Ss are being asked a numerical question about the 'whole'. The reference or standard is static and so is the length or region being measured. The reference is assigned a numerical value, usually one, and a multiplicative process of some sort is needed to make a non-random estimate. The following studies which view estimation somewhat

in this light, are concerned with quantities of containers, complexity of irregular shapes and numerousness.

Halford (1969) concluded from his investigation of the ability of children six to eight years (N=208) to judge quantities of containers that Ss construct a classificatory system corresponding to the conjunctions of heights and breadths, thus securing logical criteria for making judgments.

Siegel (1968) studied the role of experience in judgments of complexity of irregular polygons. The improvement in the processing ability of college students occurred primarily at the lower levels of complexity. However, third and sixth grade groups showed no significant improvement in performance after systematic training in discrimination at various levels of complexity.

Granberg and Aboud (1969) defined judgments of numerousness as "estimates of the number of objects in a given situation, without actually counting the objects" (p. 221). They successfully predicted that numerical estimates of an array of black dots would be affected by the number of outlined dots forming the background. The Ss were forty-eight college students. Working also with college students, Bevan, Maier and Helson (1963) found likewise that the estimates of the number of beans in a jar were consistently greater with larger containers.

The discrepancy between pupils' familiarity with the table of metric measures and their lack of knowledge of length in practical situations led the Russian educator, Sergeevich (1956), to investigate

children's judgments of magnitude, length and time. He analysed the way in which children learn to measure in metric units. He concluded that the ability to estimate accurately depends on the development of precise mental images which are consolidated only after a great many muscular and visual measurements.

Clearly there are many questions about estimation that need answers. What is the role of experience during the pre-operational and concrete-operational stages in the development of the ability to estimate? Is mathematical ability enhanced by estimation ability which has been developed initially through physical experience? What instructional procedures will lead to greater success in the estimation of answers to mathematical problems? Does a realistic confidence in the ability to estimate improve student attitude to mathematics? Is there a relationship between estimation and the acquisition of first-order conservations?

IV. CONCLUSION

Three ability domains related to mathematics learning, namely conservation, perceptual efficiency and estimation, have been the subject of this literature review. Though dealt with separately, these clusters of abilities are not completely isolated. In fact, throughout this chapter there have been running three themes which reinforce the underlying framework of this study. A brief overview of these connecting themes of perceptual constancy and conceptual invariance, spatial concepts and a multi-variate approach, effectively summarizes the literature background of this research project whose ultimate

concern is the mathematical learning of children who now lack success. First there is the matter of perceptual constancies and conceptual invariance in the face of changes in appearance, or transformations of the mental images of an object or event. Hyde (1970) asked "Are they two sides of the one coin?" (p. 39). Piaget claims that the concept of invariance is necessary for a child's development, and in particular for any real learning to occur in mathematics. If it is so essential, what is its relationship to mathematics education, not just for the school beginner, but also the children older than the bulk of the subjects in the reported research?

Spatial ideas provide the second theme. Conservation of area and interior volume involves two- and three-dimensional co-ordinated space. Estimation, when based on the physical experience of relationships between quantities, is concerned with occupied space, whether of one, two or three dimensions. The stimuli for tests of perceptual outcomes are most often spatial in nature. Evidence of a child's mental functioning can be derived from his ability to manipulate images of geometric figures internally before drawing them overtly. The Russians recognize that children need to "compare, combine or transform the figures and their elements or to examine a single object from various points of view" (Yakimanskaya, 1962, p. 148), and that a lack of flexible spatial images hinders pupils from mastering geometry (p. 152).

Though geometry is included in a number of contemporary curricula, little is known of the contribution spatial relations can

make to a child's mathematical development.

The third recurring theme is the emphasis on the multi-variate approach. Research on conservation has concentrated on single properties. Single variate training studies have met with little measurable success. By contrast, some studies in which a multi-variate approach was used have produced more favorable results. Young (1969) designed a very broad training program which successfully induced conservation in four areas. Because of the marked interaction between variables, she recommended further studies incorporating the multi-variate approach. Many studies have been directed at specific aspects of children's perceptual and cognitive development. Little has been done to synthesize the knowledge so gained in relation to mathematics education. This investigation of the contributory abilities of conservation, perceptual efficiency and estimation utilizes a spatial motif as a unifying theme within the multivariate approach. Chapter III contains the detailed description of how this motif was operationalized in the present study.

CHAPTER III

THE EXPERIMENTAL DESIGN

I. DESIGN OF THE STUDY

The main purposes of this study were to investigate the presence in children of conservation, perceptual efficiency and estimation, and to determine the relationship of these domains to mathematics achievement. The particular aspects explored for conservation were area and interior volume. For perceptual efficiency, the abilities tested were discrimination, memory and representation, in each of the visual, auditory and tactual modalities. Estimation focused on length, area and numerical quantity. A battery of tests were assembled to measure these abilities. A sample of ninety Grade 3 children was chosen so that there would be equal distribution over high, middle and low ranges of mathematics achievement.

II. THE SAMPLE

The Edmonton Separate School Board was asked to make four schools, each with two Grade 3 classes, available for data collection. The schools nominated were in four different areas of the city. This

selection of schools was fortuitous and not random. The mathematics sections of the Canadian Test of Basic Skills at the appropriate level were administered to all the Grade 3 pupils in the eight classes. The scores, in grade levels, ranged from 1.6 to 5.5, with 3.5/3.6 being the midpoint. The total range was symmetrically segmented as shown in Figure 1. The intervals of .4 between the critical segments were to insure a difference between groups. A table of random numbers was used to select thirty subjects for each of the three groups, high, middle and low. Each of the three groups was different on the Scheffe Multiple comparison of means test with a probability of less than .0001.

Socio-economic status did not appear to be a critical factor separating the groups. Analysis of the ratings on the Blishen Occupational Class Scale indicated that the three groups were drawn from populations which were not different with regard to this factor.

Table I shows the breakdown of the sample with respect to mathematics intelligence, reading and age.

III. INSTRUMENTATION

There are two main hypotheses stated in Chapter I.

(a) There is no significant difference between the scores obtained by the high, middle and low achievers in mathematics on tests

TABLE 1

MEAN MATHEMATICS, READING AND INTELLIGENCE SCORES AND AGES OF THE SAMPLE (N=90)

	High Group (N=30)		Middle Group (N=30)		Low Group (N=30)		Total Group (N=90)	
	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
Mathematics	4.7	.3	3.6	.1	2.5	.3	3.6	.9
Reading	4.8	.9	3.7	1.0	2.6	.8	3.7	1.3
Intelligence	116.3	6.7	106.3	11.7	90.5	12.5	104.4	15.0
Age (months)	104.9	3.5	105.6	5.4	105.7	5.0	105.4	4.6

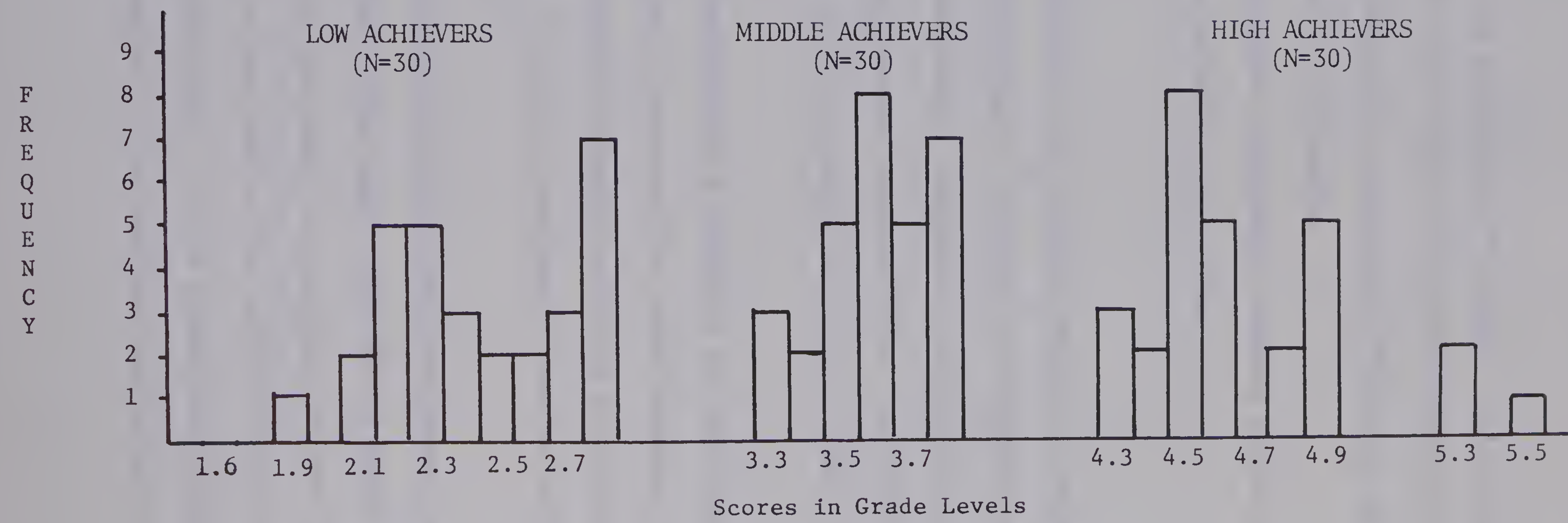


FIGURE 1

FREQUENCY DISTRIBUTION OF MATHEMATICS ACHIEVEMENT SCORES
FOR THE TOTAL SAMPLE (N=90)

of (i) conservation; (ii) perceptual efficiency; (iii) estimation.

(b) There is no significant correlation between scores obtained on tests of conservation, perceptual efficiency, and estimation and mathematics achievement.

In order to test these hypotheses, measures on the test instruments outlined in Table II were obtained.

1. CONSERVATION

The tests incorporated the Piagetian method of observing and questioning children individually. Although the Piagetian method of questioning is not a standardized technique, the type of question asked to elicit the conservation response is not a significant source of variance (Cathcart, 1969; Pratoomaj and Johnson, 1966; Mermelstein and Shulman, 1967).

In this study considerable flexibility was employed in order to ensure that the subjects understood the verbal meaning of the questions. Neutral but supportive comments were made by the investigator in order to encourage the child to continue his active participation in the test situation.

Questions eliciting rationalizations were varied, for example, "What makes you think so?" "How come?" "How do you know?" After each transformation, the child was asked the relevant criterion question, such as, "Do you have as much room in your backyard as in mine, or do you have more or do I have more?", in the Transformed Triangles Test. The order of choices implied in the questions was varied to avoid perseveration.

TABLE II

TESTS ADMINISTERED

Conservation 1. Area

2. Interior Volume

Perceptual Efficiency

MODALITY		PHASE: Discrimination	PHASE: Memory	PHASE: Representation
	Visual	Huelsman-Word Discrimination Test Form B-Alta	Test of Visual Memory	Test of Visual Representation
	Auditory	Wepman Auditory Discrimination Test	Test of Auditory Memory	Auditory Closure Test
	Tactual	Test of Tactual Discrimination	Test of Tactual Memory	Test of Tactual Representation

Estimation
1. Numerical quantity
2. Length
3. Area
4. Area-Number

Standardized Tests
1. Canadian Test of Basic Skills - Mathematics Sections
2. California Short-form Test of Mental Maturity Level 1H
3. Gates-MacGinitie Reading Test

The children were categorized as conservers, partial conservers or non-conservers. In order to quantify these categories, conservers were given a score of two, partial conservers one and non-conservers zero.

a. Conservation of area

Item 1. The Barns Test.

This test replicated Piaget's investigation into a child's understanding of the invariance of an area relationship upon subtraction of equal regions (Piaget et al., 1960, p. 273). Two pieces of card demonstrably equal in area represented two fields in each of which a cow is pasturing. One-inch wooden cubes represented barns which reduce the amount of pasture available to the cows.

Barns were placed on each field, one at a time so that the number of barns was always equal. The criterion question was, "Does this cow have as much to eat as that one, or does this one have more to eat or does that one have more to eat?" The investigator indicated appropriately when the criterion question was asked after the addition of each pair of barns.

The investigator checked with the subject that the two pieces of card were the same size, and therefore that the two cows would have the same amount to eat. Initially both sets of barns were scattered, followed by the contiguous/scattered positions, as illustrated in Figure 2. If a subject rejected the equality of area when the barns were in the contiguous/scattered positions, the two sets of blocks were positioned according to the same arrangement.

A subject was regarded as a conserver only if he could explain

the equality of the two pastures when eight pairs of barns had been placed in the contiguous/scattered position. Similarly the partial conserver, as well as the non-conserver, was judged on his response when the barns were in the contiguous/scattered position.

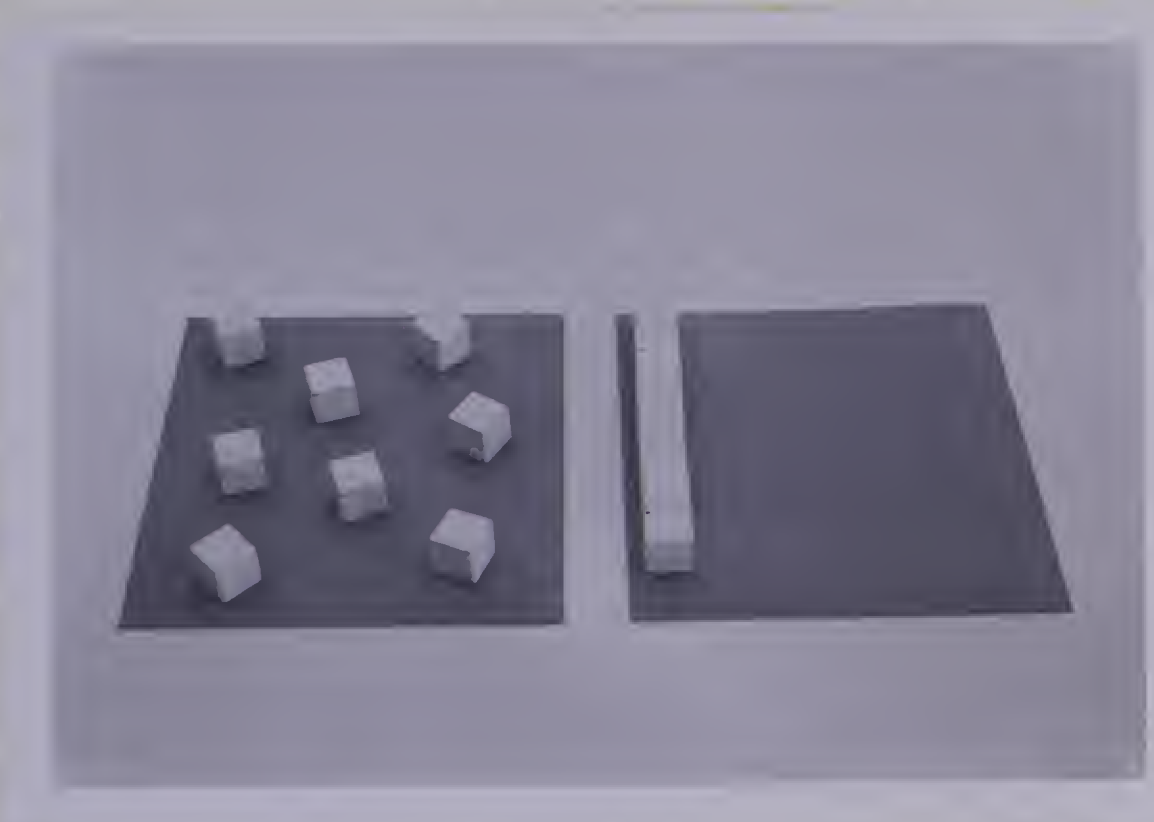


FIGURE 2

BARNS TEST: CONTIGUOUS AND SCATTERED POSITIONS OF THE BLOCKS

Item 2. The Transformed Triangles Test.

This test is also a replication of a Piagetian investigation into the conservation of area upon transformation. The child must ignore the increase in perimeter in order to acknowledge the equality of area (Piaget et al., 1960, p. 273). Two congruent triangles made from a rectangular piece of card were rearranged into a large triangle

and other shapes such as a parallelogram in order to check a child's ability to withstand the apparent change in size (p. 274).

The materials used were wooden blocks (a six-inch square, two congruent right-angled triangular blocks which together form a six-inch square, two oblongs six inches long and three inches wide). The shapes are shown in Figure 3. The subject was presented with the triangles -- "your backyard". The investigator had the square -- "my backyard". The criterion question was "Do you have as much room in your backyard as in mine, or do you have more or do I have more?" If the answer indicated that the regions were different in area, a second question was asked. "If you had to plant your backyard with grass and mine with grass, would you need as much grass for yours, or for mine, or would they both need just as much?"

The triangular blocks were rearranged first as a large triangle and then as a parallelogram. The oblongs in the form of a square represented the backyard belonging to the subject's teacher who was named. These were compared with the square and then with the triangles.

The children were categorized as conservers, partial conservers and non-conservers.

B. Conservation of interior volume

Item 1.

In order to test the conservation of interior volume, a task involving three-dimensional materials was designed to parallel Piaget's Transformed Triangles Test which looked at two-dimensional space. In the three-dimensional situation, the conserving child has to withstand

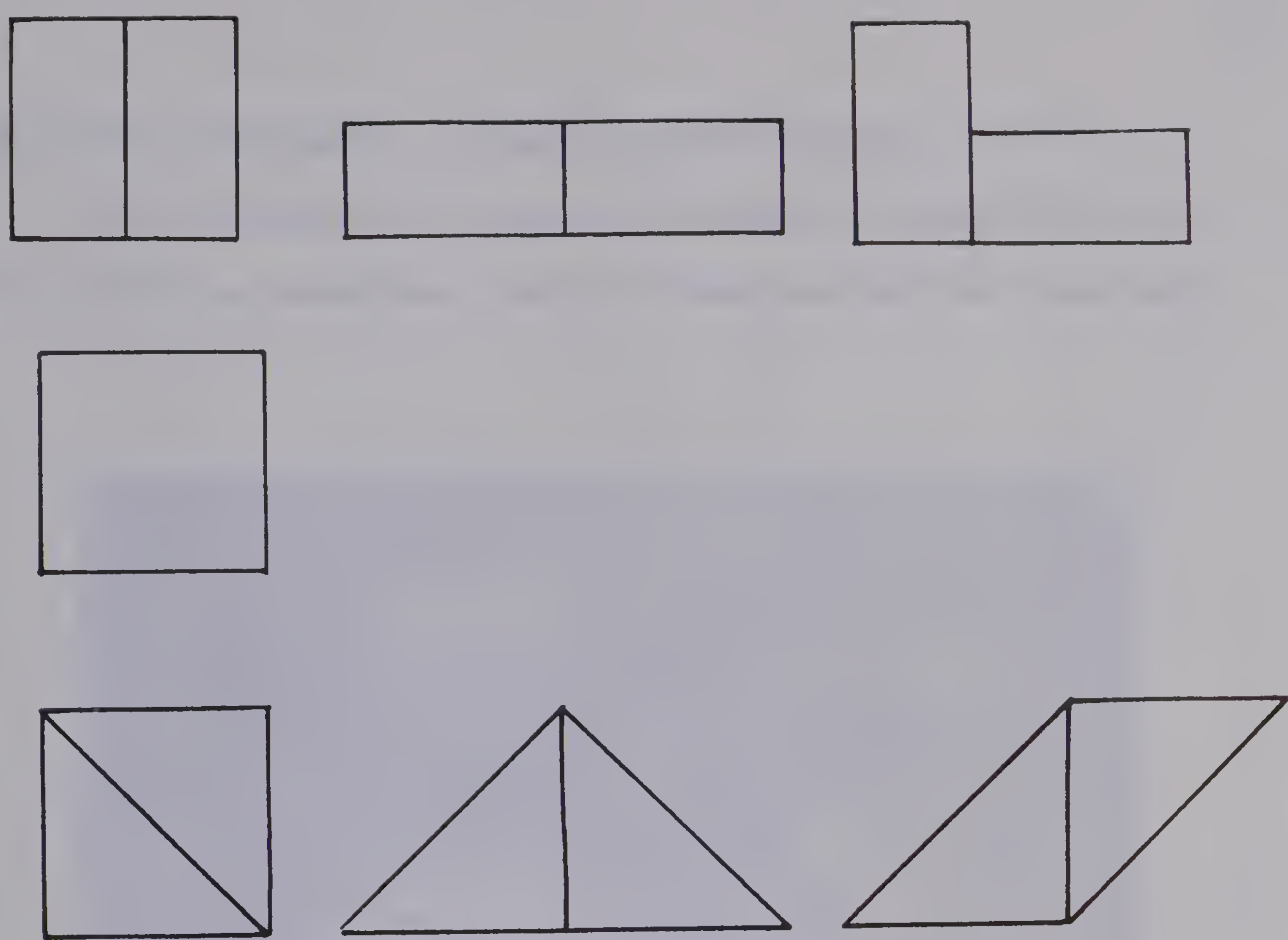
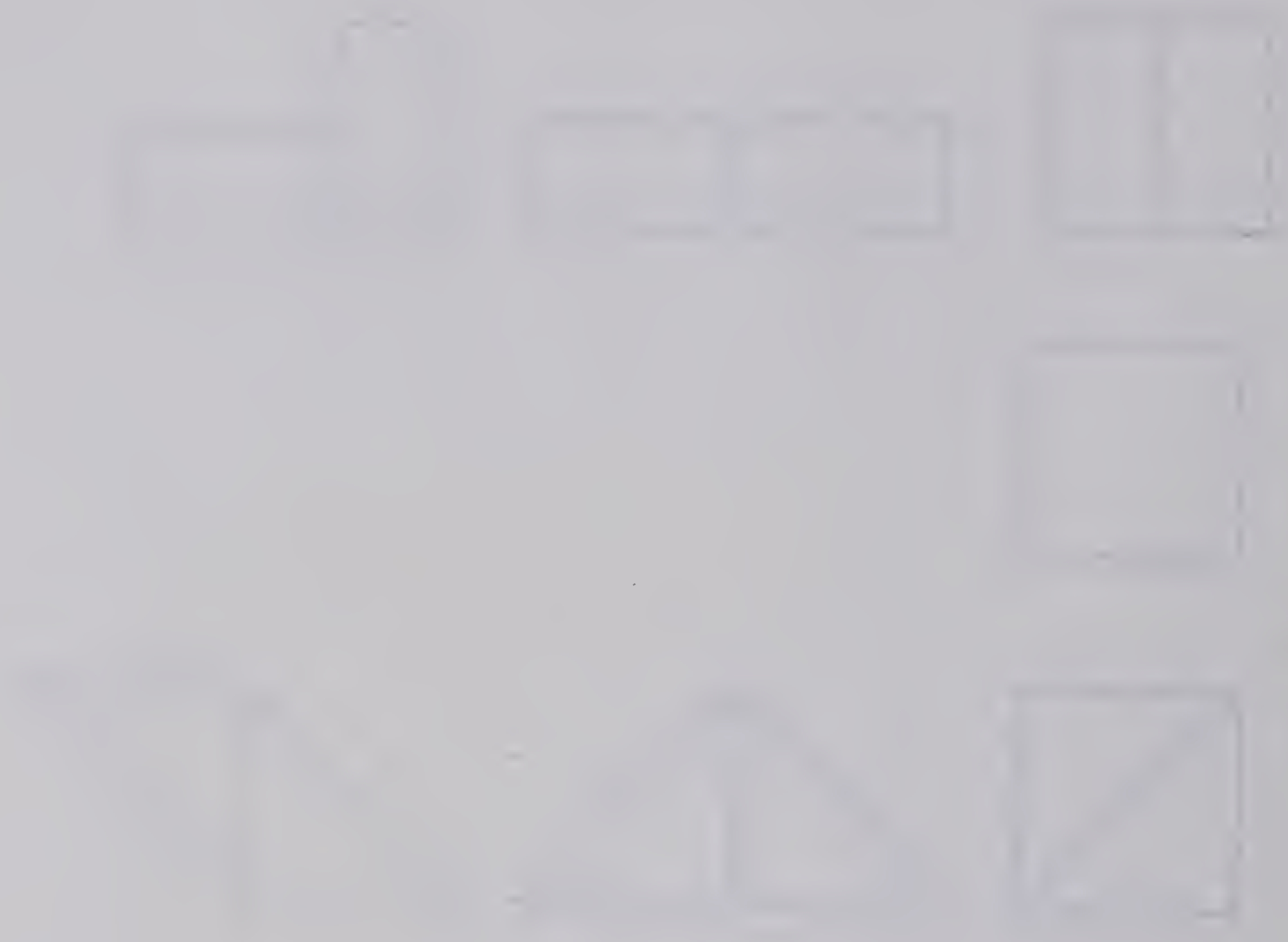


FIGURE 3

ARRANGEMENT OF THE BLOCKS FOR
COMPARISON OF AREA

the apparent increase in the size of the region occupied by the spread-out blocks.

A solid three-inch wooden cube was compared to a cube constructed of four pieces, each three inches in height but varying in the shape of their base, as illustrated in Figure 4. The four pieces were successively rearranged so that the base became increased gradually. The criterion question was "Is this (E indicating the totality of the solid block) as big as this (E indicating the totality of the four pieces together), or is this bigger, or is that bigger?" If the answer was incorrect, a second situation was posed. "Let's pretend



that this is chocolate. Is there as much chocolate . . .?"

The children were categorized, as with the other tests in this section, as conservers, partial conservers and non-conservers.

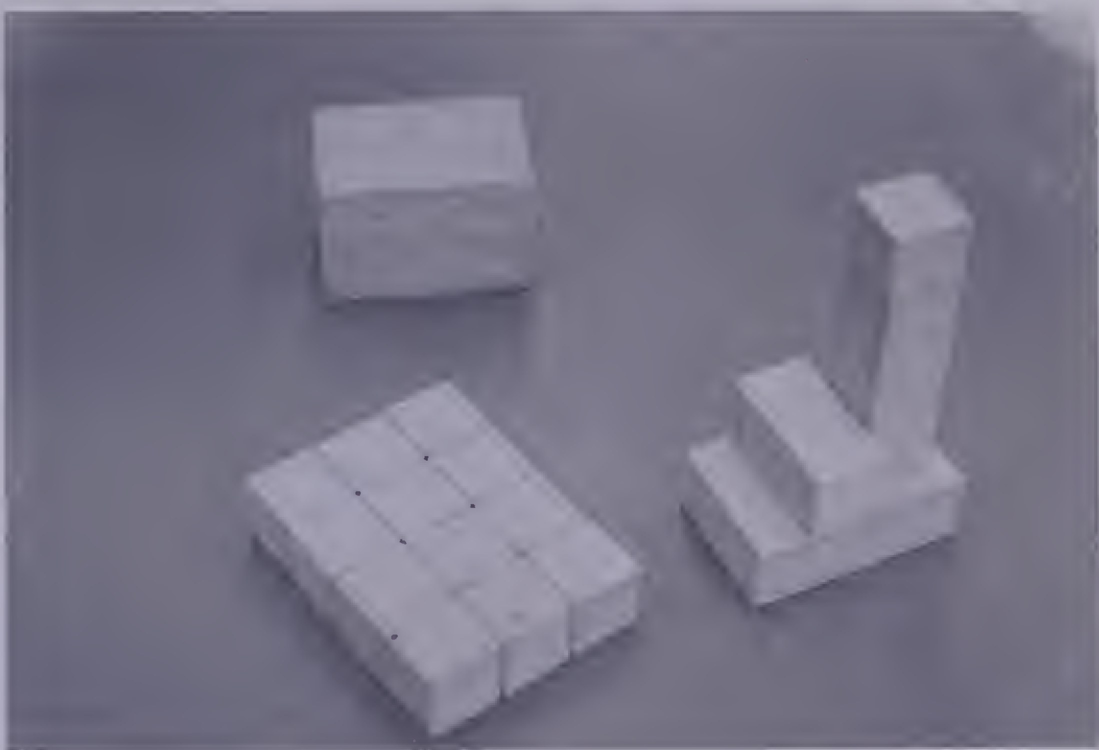


FIGURE 4

ARRANGEMENT OF THE BLOCKS FOR COMPARISON OF INTERIOR VOLUME (ITEM 1)

Item 2.

In the classic Piagetian test for interior volume, various constructions were rebuilt from an original model of thirty-six unit cubes, either by the subject or by the experimenter. The storyline told of lake dwellers whose houses were rebuilt on different sized islands after a storm (Piaget <u>et al</u>., 1960, p. 356-357). Because

the children in the present study are citydwellers, the storyline of the Islands Test was changed to the erection of apartment buildings in an urban setting. The criterion question was "Is there as much room in this apartment building as in that, or does this have more, or do they both have the same amount of room?" Twelve wooden cubes were arranged initially as a rectangular prism congruent to a single block (3x2x2). The unit cubes were successively rearranged row by row till they were flat on the table (4x3x1).

A set of blocks (one each of 3x2x1, 3x1x1, 2x1x1, 1x1x1) initially arranged as a rectangular prism (3x2x2) was compared to the single block. This was removed once the subject agreed that equality of room existed. The set of blocks was next compared to the twelve unit-cubes in an identical solid arrangement. The unit-cubes were again successively rearranged to become a flat shape (4x3x1), while the set of blocks was rearranged to make a tall building gradually. The arrangement of the wooden blocks in their final position is shown in Figure 5.

Once more, the children were categorized as conservers, partial conservers or non-conservers.

2. PERCEPTUAL EFFICIENCY

Tests from a variety of sources were selected. Some were standardized; others constructed by the researcher. Some were given to groups of subjects, others individually. Table III indicates the categories into which each test fits.

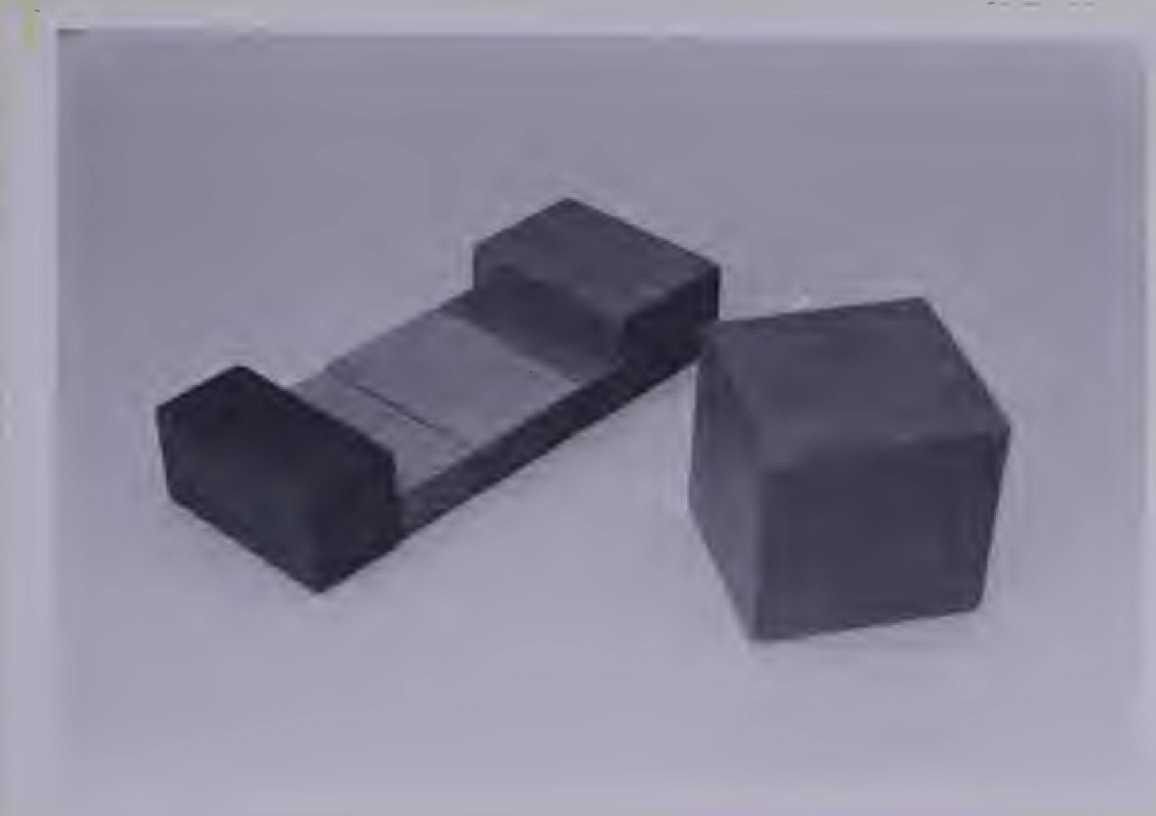


FIGURE 5

ARRANGEMENT OF THE WOODEN PIECES IN
THEIR FINAL POSITION (CONSERVATION
OF INTERIOR VOLUME, ITEM 2).

TABLE III

TESTS OF PERCEPTUAL EFFICIENCY

		PHASE		
		Discrimination	Memory	Representation
MODALITY	Visual	Huelsman-Word+ Discrimination Test Form B - Alta	Test of* Visual Memory	Test of+ Visual Representation
	Auditory	Wepman Auditory* Discrimination Test	Test of* Auditory Memory	Auditory* Closure Test
	Tactual	Test of* Tactual Discrimination	Test of* Tactual Memory	Test of* Tactual Representation

+Group test

*Individual test

If mathematics curricula more appropriate to the learning styles of some individuals are to be devised, more data are needed on how children process sensory input in classrooms now. No matter the teaching style, instruction currently depends on the auditory channel mainly, and entirely on the three modalities, visual, auditory and tactual.

Along each sensory channel there are various phases when information is processed. Three of these concern discrimination, memory and representation. Direct observation is impossible. Hence the purpose of this battery of nine tests is to study the overt responses of children to tasks involving discrimination, memory and representation as they occur in the three modalities.

a. Huelsman-Word Discrimination Test Form B-Alta.

This pencil and paper group test is an adaption of the Huelsman Word Discrimination Test which consists of nonsense words, whereas the Form B-Alta version supplies the subject with an English word to which he may refer. The child selects from a choice of five sets of letters that group which matches the reference word. The test has a two minute time limit. Each correct item is given a score of one.

The adapted Form B-Alta is used in the Reading and Language Centre of The University of Alberta (Dr. W. Fagan, Director), and is regarded as a satisfactory test of visual discrimination whether administered individually or in groups.

b. Test of Visual Memory

A sub-test of the Illinois Test of Psycholinguistic Abilities deals with visual sequential memory. Materials include a set of white plastic tiles on which black letter-like but meaningless forms are impressed. The task requires the child to duplicate the order of a sequence of geometric designs presented by the examiner and then removed (Kirk, McCarthy and Kirk, 1968, p. 27-32).

The Seventh Mental Measurement Yearbook (Buros, 1972, p. 815-825) includes a long list of studies related to the Illinois Test of Psycholinguistic Abilities as well as two reviews. According to Chase (p. 824) the twelve sub-tests "appear to be reasonably reliable at each age level". Criticisms by Carroll (p. 822) are directed mainly at the standard English expectation and the assumption of an experiential background characteristic of middle class culture. The meaningless designs on the tiles used in the Sequential Memory sub-test are not subject to such biases. Therefore this subtest appears to avoid Carroll's criticisms. Chase (p. 824) concludes that the tests have been carefully constructed and are a useful tool for the psychometrist.

The Sequential Memory subtest was adapted for the present study because the ability being considered was memory, without the complication of sequence.

A set of eighteen cards was prepared. Each of the first five showed one form only. These were used as practice examples. The test proper began with five cards each showing a pair of designs, followed

by a set of five cards with three forms on each. The order of presentation was randomized within each of these three groups. The final three cards showed four, five and six designs respectively. Each card was presented to the individual subject for three seconds. The time was measured from when the child actually looked at the card. He was required to duplicate the designs from the set of tiles, but not the order.

For each design correctly selected, the subject received one point.

c. Test of Visual Representation

This group test was based on a Piagetian experiment, in which children were required to copy models of topological and euclidean shapes (Piaget and Inhelder, 1967, p. 53). In the present study the children copied eight models by drawing. Five of the models came directly from Piaget and Inhelder's set of twenty-one shapes (p. 54), and appear in Riggs' (1972) Test of Euclidean Space (p. 174-75). Three of the items were taken from his Test of Topological Space (p. 171). A triangle on a vertex rather than on a base was also included as Graham, Berman and Ernhart (1960) had done (see Chapter II, p. 29). The models were drawn boldly on large cards (ten inches long and eight inches wide) and are shown in Figure 6.

An extra factor was included. The children were required to rotate their drawings through a quarter turn, except for one triangle which was rotated through a half turn. In the practice situation, two children faced each other. The group was asked "What would A draw

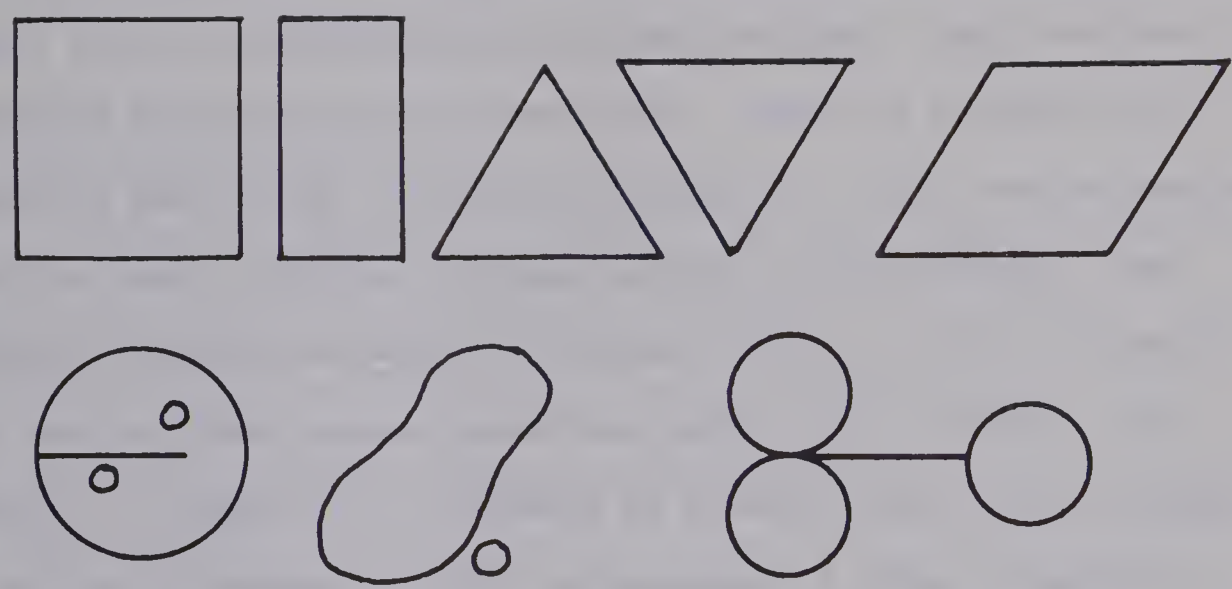


FIGURE 6

TEST OF VISUAL REPRESENTATION:
EUCLIDEAN AND TOPOLOGICAL MODELS

looking at B?" "Without moving to look, how would A draw B if he stood behind B?" In the test situation the investigator showed a toy seal looking at one of the geometric figures, the same way as the children. The seal was moved to the rotated position. "Draw what the seal is seeing." At the bottom of each of their eight oblong sheets of paper stapled into a vertical booklet, the children drew a seal's face to clarify further the task required. The cards were presented horizontally in random order. After the children had completed their first and second drawings, the cards were rotated so that the seal's view was the same as that of the children. The scoring procedure was taken from Riggs (1972, p. 171-75). Each of the five euclidean shapes gained one point for (i) closure, (ii) presence of angles, (iii) presence of straight lines, (iv) accuracy of angles (tolerance = $\pm$ 10°), and accuracy of lines (tolerance = $\pm \frac{1}{5}$ by proportion). The three topological models gained one point for (i) closure, (ii) separation, and (iii) proximity with the same degree of tolerance as above. Total possible score is thirty-three.

d. <u>Wepman Auditory Discrimination Test</u>

The stated purpose of this test is "to determine a child's ability to recognize the fine differences that exist between the phonemes used in English speech" (Wepman, 1958, p. 1). The test consists of thirty pairs of words differing in a single phoneme in each pair, plus ten word pairs which do not differ to serve as false choices. According to Walby (1965), this test appeared to be the most adequate instrument available. It is recommended highly as an

accurate test of auditory discrimination by Di Carlo (Buros, 1965, p. 940-1).

The test was administered according to the instructions set out in the manual of directions (Wepman, 1958). The test was scored by counting the number of times the child said the words in a pair were the same when they were actually different (p. 2). The test-retest administration showed a reliability of .91 (p. 3).

e. Test of Auditory Memory

This test consisted of twenty-five sentences, the first being just one word, but getting progressively longer. The subjects attempted to repeat the sentences which were presented on a tape recorder. This test was devised by Arthur Benton, a neurologist from Hospital Iowa City, Iowa, whose recording of the sentences was used in this study. Knights (1966) collected normative data on this and the following test.

In the present study, no feedback was provided, However the investigator made supportive sounds. If a subject was obviously perplexed, the investigator said, "Let's try the next one." This test was scored according to the procedure set out in the Examiner's Handbook for the Detroit Tests of Learning Aptitude (Baker and Leland, 1967). A sentence with no errors scored three points, with one error two points and with two errors one point.

An alternate method of scoring, suggested by Knights (1966) allowed one point only if the entire sentence was correct (p. 3). A second score, using this procedure, was obtained for each child. A

factor of approximately 3.43 related the two scoring methods when applied to the total sample as well as to each of the three mathematics achievement groups.

The Auditory Closure Test and the Test of Auditory Memory are used in the Neuropsychological Clinic of The University of Alberta, and are regarded as satisfactory instruments to measure the two auditory abilities.

f. Auditory Closure Test

This test was adapted by Knights (1966) from the Auditory Closure Test of Corinne Kass who carried out research work under the senior author of the Illinois Test of Psycholinguistic Abilities. The subjects attempted to blend into words progressively longer chains of sound elements. The twenty-three items, presented on a tape recorder, had been selected because of their high familiarity as words for children (Knights, 1966, p. 3).

The score was the correct number of responses.

g. Test of Tactual Discrimination

This individual test was taken from Page (1959), who attempted to repeat Piaget and Inhelder's (1967) experiment reported under the heading "Haptic Perception" (p. 19). Page, whose study is the only one of its kind mentioned by Flavell (1963, p. 389),acknowledged that

> as Piaget and Inhelder do not set out precise details of the apparatus used and the conditions of testing, it was not possible to ensure that identical tests were given this time, although the original experiments were followed as closely as possible (Page, 1959, p. 16).

In their description of the original investigation, Piaget and Inhelder (1967) had the child feel "a series of cardboard cut-outs in the shape of geometrical figures", listed as

a. simple and symmetrical

b. more complex but symmetrical

c. assymmetrical, but with straight sides

d. a number of purely topological forms (p. 19).

Page (1959) reproduced his interpretation of these four sets of shapes. For the present study eight of these shapes were selected. The six simple euclidean forms of Set C plus the two stars from Set B were magnified six-fold and cut out of stout cardboard. The eight shapes resulting were put into two groups, as shown in Figure 7, so that each set of stimulus and choice shapes was independent. The subject felt a cardboard shape in one fabric bag, then attempted to select the identical shape from a choice of four out of sight in another bag. He was free to refer to the hidden stimulus shape if he wished. The sets were alternated. Within each set the order was randomized. At no time in the entire investigation did any subject see any of these cardboard shapes.

The score was the total number of correct responses.

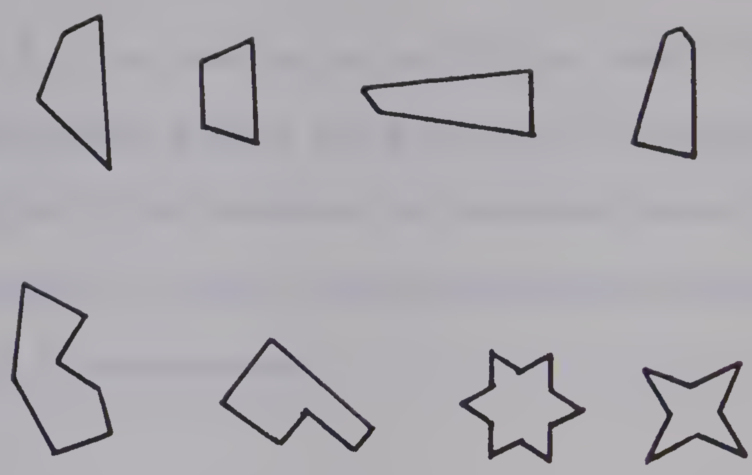


FIGURE 7

TESTS OF TACTUAL DISCRIMINATION AND MEMORY

h. Test of Tactual Memory

This test was identical to the Test of Tactual Discrimination, except for two conditions. First, the subject was not permitted to refer back to the stimulus shape. Secondly, starting at one hundred, the subject counted a decade backwards between feeling the stimulus shape and seeking its match in the other bag.

The score was the total number of correct responses.

i. Test of Tactual Representation

Eight shapes made of pineboard were placed singly in fabric bags, so that they were out of sight. The subject was required to feel and draw each shape. He was free to feel the shape in the bag. Each of the shapes was part of a six-inch square or a six-inch equilateral triangle. All the shapes had a three-inch base, except the isosceles triangle which had a six-inch base. The shapes are

shown in Figure 8. The scoring procedure was taken from Riggs (1972, p. 174-75). Each shape gained one point for (i) closure, (ii) presence of angles, (iii) presence of straight lines, (iv) accuracy of angles (tolerance = $\pm$ 10°), and (v) accuracy of straight lines (tolerance = $\pm \frac{1}{5}$ by proportion).

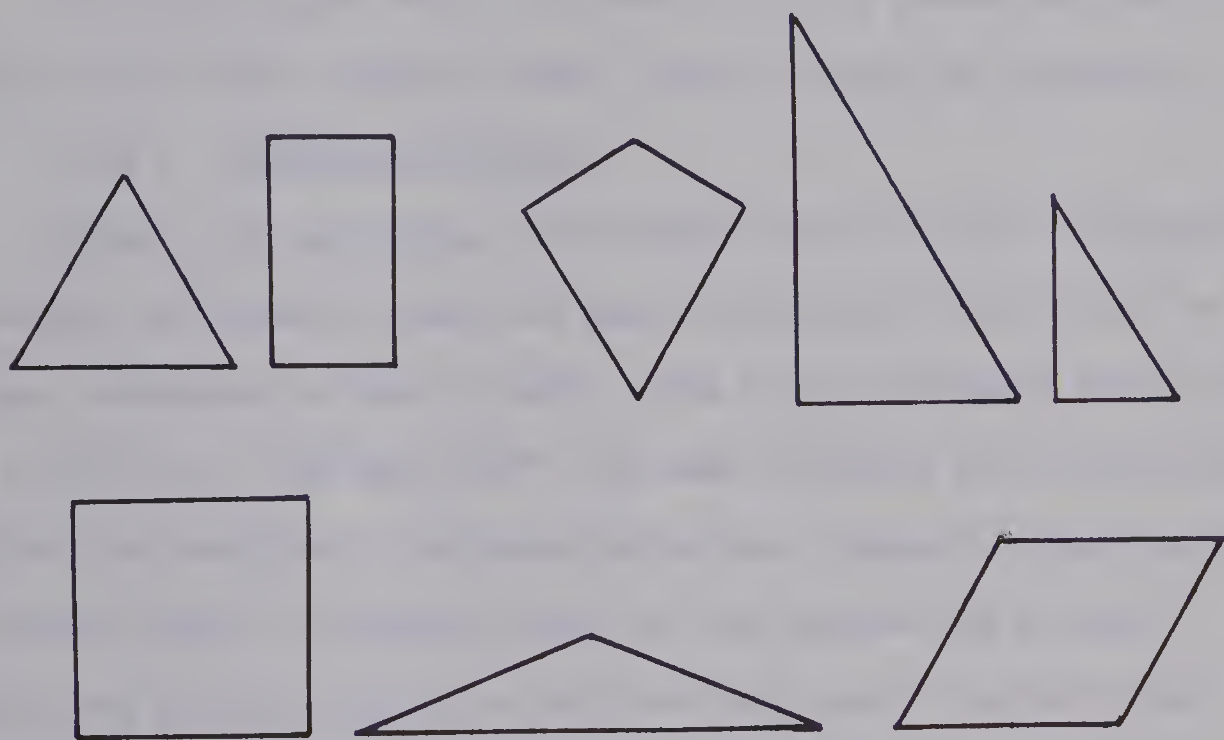


FIGURE 8

MODELS FOR THE TEST OF TACTUAL REPRESENTATION

To check the reliability of the researcher's classification of student responses on this and the Test of Visual Representation, a second judge classified the drawings of a random sample of twenty-eight subjects. An inter-judge reliability of .985 resulted.

3. ESTIMATION

In this group of tasks, the ability of a child to estimate numerical quantity, length, area, and area-number was being tested. Only the first test, of numerical quantity, was administered individually. In the other three tests, the children recorded their responses on paper.

In scoring these tests a tolerance of $\frac{1}{5}$ by proportion was allowed in each case, whether number, length or area was involved.

Item 1. Numerical Quantity

Shown a jar containing sixty marbles (four layers of fifteen) the subject was asked to guess how many marbles were in the jar. He was next instructed to take a handful from a bag of marbles and place them in the jar. "How many now?" The same procedure was followed for a further two handfuls. The investigator then removed a large handful so that the number of marbles in the jar was between the original quantity and the quantity after the first addition. Once more the subject was asked to make an estimate. This test was given individually.

As the number of marbles in the jar was sixty, an estimate of forty-five through seventy-five scored one point. One "handful" of marbles is approximately ten, therefore the second estimate had to be seven through thirteen more than the first. The third estimate gained a point if double the first handful, with the same tolerance of $\frac{1}{5}$, was added to the second estimate. The final estimate was successful only if it came between the first and second estimates. The investigator

carefully observed the size of both the "handful" and the "double handful". If a child did not pick up approximately the appropriate number of marbles, a second instruction to make sure he had a "good" handful or double handful was added.

Total score possible was four points.

Item 2. <u>Length Estimation</u>

The materials for this group test were six-inch and eight-inch strips of inch wide cardboard of various colours, and a four-foot strip of inch-wide white cardboard which was place horizontally on the window ledge of the classroom. The blackboard was in most cases four feet high. Where it was not, a point forty-eight inches above the bottom of the blackboard was marked. The children were asked, "How many of these (E holding the appropriate strip either vertically or horizontally) would fit along the strip (or up the blackboard)?" The children recorded their estimates on an answer sheet provided. The order as set out in Table IV was the same for all subjects. The difference between the child's estimate and the true answer on each of the eight tasks was added to give a total. This result was classified according to the following five-point scale.

Total Differences	5	6-11	12-16	17-21	22
Scaled Scores	4	3	2	1	0

Item 3. <u>Area Estimation</u>

The surface being estimated was the top of a standard Grade 3 desk. Where a child was not sitting at such a desk the standard desks in the classroom were pointed out, so that each subject's attention

TABLE IV

LENGTH ESTIMATION: ORDER OF TASKS

		6" Strip		8" Strip	
		Vertical	Horizontal	Vertical	Horizontal
Four Foot Length	Vertical	1	4	6	8
	Horizontal	7	3	2	5

was focused on a correct surface. The children were shown a pineboard shape and asked, "How many of these do you think would fit on your desk, without any spaces or overlapping of the edges?" Estimates were recorded on the answer sheet provided. The reference shape was in sight during each task. The four reference shapes, each related in area to a six-inch equilateral triangle, are shown in Figure 9.

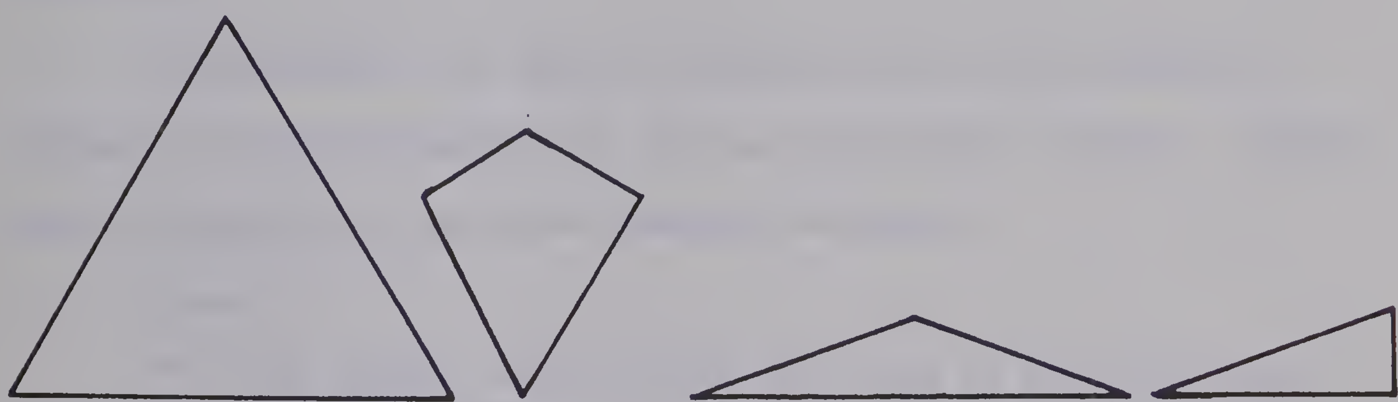


FIGURE 9

AREA ESTIMATION: REFERENCE SHAPES

As the correct responses for the four items were fifty-one, fifty-one, seventeen and one hundred and two, estimates which fell in the ranges thirty-nine through sixty-three, thirty-nine through sixty-three, thirteen through twenty-one, and seventy-six through one hundred and twenty-six respectively gained one point each. Total possible score was four.

Item 4. Area-number Estimation

This group test incorporated the estimation of area within a multiplicative process. In the practice situation the investigator informed the group that a value of three was assigned to a window pane. She asked, "What is the value of this window pane?" . . . "What is this row of window panes worth?" In each case the investigator indicated by gesture that surface area was the property being considered.

Unfortunately the eight classrooms did not have identical windows, so some differences in the test situations occurred. However the relationships in each case remained unaltered.

Task 1.

"If this window pane is worth five, what is this lot (indicating a set of four) worth?"

Task 2.

Two oblongs, one four times as large as the other, were drawn approximately one foot apart on the blackboard. "If this oblong (E indicating the smaller) is worth twelve, what is the value of the other one?"

Task 3.

A small oblong was drawn on the blackboard at the front of the room, while another six times as large was drawn on a side blackboard. The children were asked to estimate the value of the larger region, when the small oblong was worth ten.

Task 4.

The children were shown two blocks, one a six-inch equilateral triangle, the other an isosceles triangle one-third its size. "If this block (E indicating the smaller block) is worth seven, what is this (indicating the larger block) worth?"

The four correct responses were twenty, forty-eight, sixty and twenty-one. Therefore the tolerable estimates were in the ranges sixteen through twenty-four, thirty-six through sixty, forty-five through seventy-five, and fifteen through twenty-six respectively. Total possible score was four.

4. STANDARDIZED TESTS

a. The Canadian Test of Basic Skills: mathematics sections.

The mathematics section of this group test has two parts. The first, Mathematical Concepts, is designed to evaluate the students' understanding of the number system, of mathematical laws and the operations. The second subtest is claimed to measure problem solving ability. By totalling the sub-scores a comprehensive mathematics score is obtained for each student.

According to Birch (Buros, 1972), the Canadian Test of Basic Skills has such a long line of respected antecedents that its status need never be in doubt. This reviewer praises the technical sophistication of test design and the production of norms. Standardization was on a group of over thirty thousand children drawn from a stratified random sample of over two hundred schools throughout Canada. Although it makes very few concessions to the newer

mathematics syllabuses, Birch maintains that "for the present, this is probably as useful an instrument as exists" (p. 6).

The test was administered by the eight classroom teachers, but all the correction was done by the investigator. The test was scored according to the procedures set out in the manual.

b. Gates-MacGinitie Reading Test Survey C

Two sections of this group test were administered as part of a system-wide survey of reading achievement. The vocabulary section which attempts to "sample the student's reading vocabulary" (Gates and MacGinitie, 1965, p. 1) is graduated in terms of familiarity and difficulty. The comprehension section attempts to measure "the student's ability to read complete prose passages with understanding" (Gates and MacGinitie, 1965, p. 1). It uses an irregular closure procedure and is also graduated in terms of difficulty. Reviewers (Buros, 1968) have concluded that the test is extremely useful for determining the level of competence of groups of pupils in reading (p. 303-307).

Scores were taken from the students' cumulative record cards held by the school.

c. California Short-Form Test of Mental Maturity, S Form, Level 1 H

This group test is composed of seven subtests which yield four factors, Logical Reasoning, Numerical Reasoning, Verbal Concepts and Memory. Sub-total scores are obtained on Language and Non-Language. The total score is labelled General Intelligence. Stanley (Buros,

1965) indicated that in general the test is very acceptable for use as a measure of mental maturity (p. 444).

The teachers administered this test to all their pupils, the corrections being done by the investigator. The test was scored according to the procedures set out in the manual.

5. BLISHEN OCCUPATIONAL CLASS SCALE

The school records were used to obtain the father's or guardian's occupation, which was categorized according to Blishen's Occupational Class Scale (Blishen et al., 1968, p. 741-753). This scale is based on an analysis of the education and income characteristics of incumbents of occupations drawn from the 1961 Canadian census.

Where the occupation information was missing, vague or unusual, the investigator had to make judgments as to similar occupations listed in the Blishen Scale. For this reason, some doubt must be cast on the accuracy of conclusions based on these data. In this study socio-economic status is not an important variable. It was included as a check that the sample was not unduly biased one way or the other.

IV. PILOT STUDY

A pilot study involving nine Grade 3 children from a school in another part of the city was conducted in March 1972. The class teacher was asked to nominate three high achievers in mathematics, three middle achievers and three low achievers. The purpose of the pilot study was to examine the suitability of the various tests for

children of this grade level. It was felt necessary to eliminate doubts arising from the review of related literature that normal children of eight and nine years, irrespective of intelligence, would all achieve success on many of the tests particularly those of perceptual efficiency. The range of scores for this small group warranted continuing with the main study.

V. DATA COLLECTION

The data were collected between mid-January and early-May, 1972. The classroom teachers administered the Gates-MacGinitie Reading Test in January, the mathematics sections of the Canadian Test of Basic Skills and the California Short-Form Test of Mental Maturity in May. Three graduate students in the field of educational psychology shared in the administration of the tests of Auditory Memory and Auditory Closure and the two tests of representation. The researcher conducted all other tests personally. For the individual tests each school provided a comfortable, well-lit and private room. All subjects seemed to participate enthusiastically in the test situations. The order of the tests was altered, but not to a particular pattern. Conditions within the schools affected choices in some instances.

Age, father's occupation and reading scores were secured from the cumulative record cards.

VI. ANALYSIS OF DATA

The data for this study were analysed using the following procedures which have been prepared as computer programs by the

Division of Educational Research Services of The University of Alberta.

(1) Pearson Product Moment Correlation (DEST 02)

Correlation matrices were computed for age, sex, reading, intelligence, conservation, perceptual efficiency, and estimation variables over mathematics groups and total sample.

(2) Partial Correlation (ANOV 10)

The effect of intelligence was partialled out from the other variables listed above.

(3) One-way Analysis of Variance (ANOV 15)

This test was used to determine whether differences existed between (i) the mathematics groups, and (ii) conservers and non-conservers, on the tests mentioned above.

(4) Scheffe Multiple Comparison of Means (ANOV 15)

This procedure was used as a comparison of means following the above analysis of variance. In this way it could be determined whether there was significant differences between the means.

(5) Two-Way Analysis of Covariance (ANOV 20)

This test was used so that interactions could be examined between mathematics, sex, conservation, perceptual efficiency and estimation.

(6) Stepwise Regression Analysis (MULR 06)

This procedure was used to discover which variables can best predict achievement in mathematics.

(7) Factor Analysis (FACTO 1)

This test was used to determine if the original set of variables could be reduced to a set of factors, amenable to interpretation.

CHAPTER IV

ANALYSIS OF THE DATA

In this chapter an evaluation of the instruments used in this study is presented. A brief overview of the results is followed by a descriptive analysis of the data pertaining to the total sample. Also are reported the reactions of individual children to the tests. The second major section includes a statistical analysis of the data with the emphasis on group relationships and differences. Correlation coefficients are shown to indicate the relationships between the major variables. The analyses discussed in this chapter were carried out on the I.B.M. 360/67 computer using Fortran on programs devised by the Division of Educational Research, of The University of Alberta.

I. OVERVIEW OF THE TEST BATTERY

It will be recalled that the ninety subjects selected on the basis of high, middle and low achievement in mathematics were administered a battery of tests in order to assess

conservation of area and interior volume

visual discrimination, memory and representation

auditory discrimination, memory and closure

tactual discrimination, memory and representation

estimation of length, area and numerical quantities

mathematics, reading and intelligence

and the relation, if any, between the variables.

The means and standard deviations of the mathematics, reading and intelligence scores and of age were given in Table I. Means and standard deviations of the performance of the total sample and the three groups on the tests measuring conservation, perceptual efficiency and estimation are presented in Table V.

II. RESULTS OF EACH TEST

A. Conservation

The Barns Test and the Transformed Triangles Test, both replications of Piagetian investigations, were used to measure conservation of area. The two items on interior volume, also derived from Piaget, used a rearranged cube and an adapted form of the Islands Test respectively.

The correlations between the tasks for the total group of ninety subjects are set out in Table VI. A correlation of .60 occurred between the two area items, while the two interior volume items had a correlation of .88. The correlation between the area items taken together and the interior volume items taken together was .73. All of the correlations shown in the table were significant at the .000001 level of probability.

Twenty-one children or twenty-three per cent of the sample were classified as conservers of both area and interior volume. The basis for inclusion in this category was classification as a conserver on all four items. These twenty-one subjects were found in the high (N = 10), middle (N = 5) and low (N = 6) mathematics achievement groups.

TABLE V

MEAN SCORES FOR CONSERVATION, PERCEPTUAL EFFICIENCY AND ESTIMATION (N=90)

	High Group (N=30)		Middle Group (N=30)		Low Group (N=30)		Total Group (N=90)	
	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD
A. <u>CONSERVATION</u>								
Area	2.1	1.7	1.8	1.7	1.4	1.7	1.8	1.7
Interior Volume	2.2	1.9	1.3	1.7	1.0	1.6	1.5	1.8
B. <u>PERCEPTUAL EFFICIENCY</u>								
Visual Discrimination	33.6	7.5	28.6	9.1	23.9	7.1	28.7	8.8
Visual Memory	25.1	2.9	24.5	3.5	23.8	2.8	24.4	3.1
Visual Representation	26.8	4.0	28.0	1.2	26.4	2.3	27.0	3.8
Auditory Discrimination	6.0	3.3	6.3	2.1	9.3	5.3	7.2	4.0
Auditory Memory	49.0	6.2	44.1	6.0	38.2	6.6	43.8	7.6
Auditory Closure	14.2	3.8	9.5	3.7	7.8	3.4	10.5	4.5
Tactual Discrimination	5.4	1.6	4.6	1.8	4.5	1.6	4.8	1.7
Tactual Memory	5.8	1.7	5.0	1.5	4.8	1.6	5.2	1.7
Tactual Representation	28.2	4.7	26.0	6.5	27.3	4.5	27.2	5.3
C. <u>ESTIMATION</u>	6.5	2.0	5.2	1.9	4.6	1.9	5.4	2.1

TABLE VI

CORRELATIONS BETWEEN CONSERVATION TESTS

		Conservation of Area			Conservation of Interior Volume			Total Conservation
		1	2	Subtotal	1	2	Subtotal	
CONSERVATION OF AREA	1		.60	.90	.49	.55	.53	.76
	2			.88	.76	.78	.79	.89
	Subtotal				.69	.73	.73	.92
CONSERVATION OF INTERIOR VOLUME	1					.88	.96	.89
	2						.96	.91
	Subtotal							.93
Total Conservation								

All significant at the .01 level of probability

a. Conservation of area

Item 1. The Barns Test.

Of the ninety subjects, forty-two were classed as conservers on this test. Their verbal responses can be classified into two broad categories. The criterion question was, "Have the cows as much to eat as each other? How do you know?" In one category were responses such as "All of them take just as much room, only these are scattered and these are in a row" (Natalie, age 9.3), or "This one covers up just as much space as this one" (Kelly, age 9.1). In the other category were responses such as "Same number" (Shane, age 8.10) or "Even numbers and they're the same" (Hanna, age 8.5). Children (N = 15) who made responses similar to those in the first category seemed to be thinking about the area covered by the barns, while those (N = 27) who made responses similar to those in the second category seemed to be thinking about the numerical equivalence of the barns in each field.

The forty-five non-conservers cannot be so clearly categorized. In one category were responses such as "More space if the barns are closer together" (Yvonne, age 9.1), or "The barns in the straight line make plenty of space and these (indicating the scattered barns) cover all the field" (Clara, age 9.2), or "These are altogether in a line and there's nothing over there"(pointing to the open space) (Angela, age 8.9). Children in this category indicated that the field with the barns in the contiguous position had the greater area unoccupied. In the second category were responses such as "These are stuck together and there are no spaces for him to go around" (Diane,

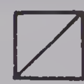
age 9.1) or "There's lots of grass here because they're spread out and these are bunched up" (Fausto, age 8.7). Children in this category indicated that the field with the barns in the scattered position had the greater area unoccupied. In a third category were responses which seemed to focus on the distance of the barns from the ends of their respective fields. Examples of such responses were "More distance from the ends and so he's got more to eat" (Cheryll, age 8.2) or "There is more space between these (the scattered barns) and the barns close together are more close to the end" (Debbie, age 9.8).

None of the non-conservers made a response that could be categorized as a numerical rationalization.

In this study the proportion of conservers on the Barns Test was .47. The mean age of the sample was 8.8 years. Though direct comparison is not possible, indirect comparison may be made with Goodnow and Bethon's (1966) results of .38 for average eight-year-olds and .65 for superior eight-year-olds on the same Barns Test.

Item 2. Transformed Triangles Test

Of the ninety subjects, thirty were classed as conservers. Their verbal responses can be classified in two broad categories. The criterion question was "Is your backyard the same size as my backyard? How do you know?" In one category were responses such as "At first it's the same and if you turn it around it's still the same" (Chris, age 8.11), or "At the first they were the same and you never took out anything" (Gladys, age 8.7). In the other

category were responses such as "If you put it together like this, it's still a square. Any shape you make it, it will make the same amount still" (Joanne, age 8.11), or "Doesn't matter if they're long or skinny, they're still the same yards with the same stuff in them" (Kelly, age 9.1), or "When they were like this () they were even, and they'd still be the same size if they're like this (). If this () is the same size as () then it's the same size as () (Cheryll, age 8.2). Children who made responses similar to those in the first category seemed to think back to the original state of the blocks after each transformation, while the children whose responses were similar to those in the second category seemed to be able to think of the transformations in succession.

Of the forty-five non-conservers, most were unable to state their reasons. The verbal responses cannot be easily classified. Two children whose rationalizations would tend to categorize them as conservers claimed that there was more unoccupied space on one or other of the fields. Their responses were, "You just keep on changing them around" (Giselle, age 8.11), and "The backyards have the same amount of grass" (Gary, age 9.0).

Figure 10 shows the numbers of conservers, partial conservers and non-conservers on the two area items. It also includes a subtotal for area conservation. To be categorized as a conserver on this combined score, a subject had to be successful on both the items. Similarly, a non-conserver was such on both of the items making up the combination. Based on these results

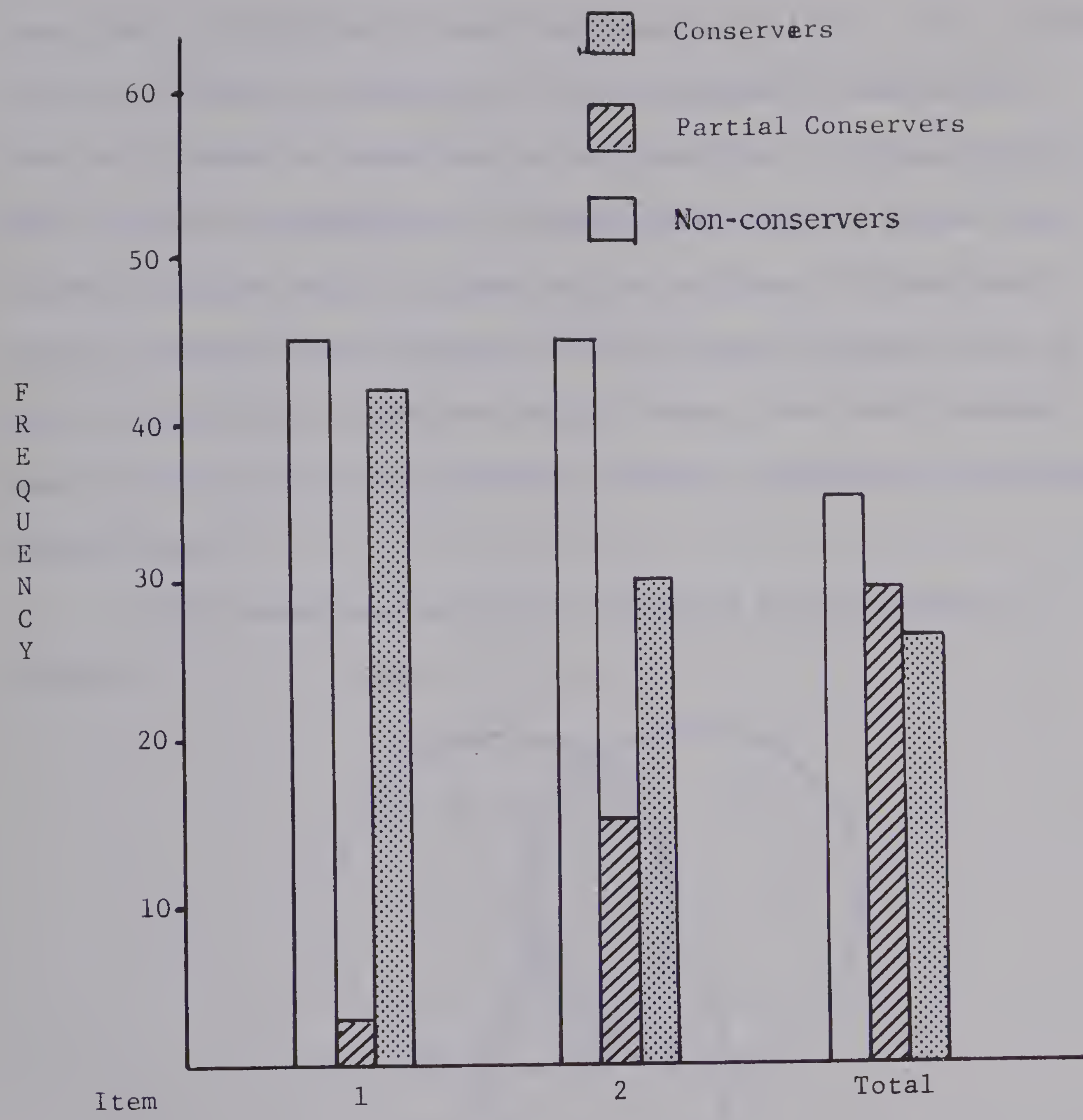
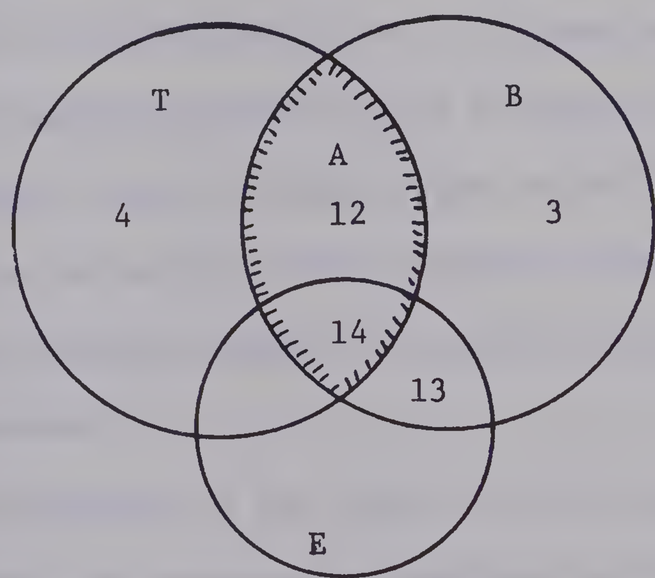


FIGURE 10

DISTRIBUTION OF SCORES ON THE TESTS
OF CONSERVATION OF AREA

twenty-six children were classed as conservers of area. Four children who were classed as conservers on the Transformed Triangles Test were not classed as conservers on the Barns Test. Sixteen children were classed as conservers on the Barns Test, but not on the Transformed Triangles Test. Of these sixteen children, thirteen gave verbal responses which indicated that they were thinking of the numerical equivalence of the two sets of barns. Into this category also fit fourteen of the twenty-six children classified as conservers on both items.

This categorization of the conservers is represented in Figure 11.



A = Conservers of Area (N = 26)
T = Conservers on the Transformed Triangles Test (N = 30)
B = Conservers on the Barns Tests (N = 42)
E = Numerical Rationalizations (N = 27)

FIGURE 11

CATEGORIZATION OF THE
CONSERVERS ON AREA TASKS

b. Conservation of interior volume.

Item 1.

Of the ninety subjects twenty-nine were classed as conservers on this test involving a rearranged cube. The criterion question was, "Is this (arrangement) as big as that? How do you know?" The verbal responses can be classified into two broad categories similar to the Transformed Triangles Test. In one category were responses such as "Because they started the same" (Gilda, age 8.6), or "When they're back like this they're still the same size" (Hanna, age 8.5). In the other category were responses such as "No matter which way you put them, they're still the same" (Mark, age 8.11) or "Altogether it's the same size. It stays as much" (Mark, age 8.3). Children who made responses similar to those in the first category seemed to go back to the original state of the blocks after each rearrangement, while the children whose responses were similar to those in the second category seemed to be able to think of the transformations in succession.

The verbal responses of the fifty-five non-conservers cannot be easily classified. For most of the children any change in arrangement seemed to imply automatically a change in size. There does emerge a category where the child seemed to focus on the surface area of the table occupied by the blocks. An example of such a response was "These (rearranged) pieces are bigger because they are lying down" (Gladys, age 8.7). Another kind of response seemed to indicate that the solod cube was thought of as being "fat and wide and so it's bigger" (Joseph, age 8.0).

Item 2.

Of the ninety subjects, thirty-one were classed as conservers on this version of the Islands Test. The criterion question was, "Is there as much room in this apartment building as in that? How do you know?" The verbal responses can be classified into two broad categories similar to the Transformed Triangles Test and the Rearranged Cube test. In one category the children's responses indicated that after each transformation they returned to the original state of the blocks. In the other category the children's responses indicated that they moved from one transformation to the next. Examples of the first category are, "Same size to start" (Gilda, age 8.6) or "At the beginning it was as big and if you put it back it will be as big again" (Cheryl, age 9.1). Examples of the second category of responses were, "Put them together and they're the same size" (Barry, age 8.3) and "You could make it the same" (Chris, age 8.11).

The fifty non-conservers did not respond according to clear categories. For most of them, any change in arrangement seemed to imply a change in size.

Lunzer (1960) used the Islands Test on only four children at each age level. His fifty per cent success at age eight can be compared with the thirty-four per cent in the present study in which the majority of the subjects were aged eight (X = 104 months). Lovell and Ogilvie (1961) classified sixty-five per cent of the age eight group as conservers on an adapted form of this test. The large discrepancies between the English experiments and the present study may

be due to variations in intelligence which the former did not report.

Figure 12 shows the numbers of conservers, partial conservers and non-conservers on the two interior volume items. It also includes a subtotal for interior volume conservation. To be categorized as a conserver on this combined score, a subject had to be classed as a conserver on each item. Similarly a non-conserver was such on each of the items making up the combination. On this basis, twenty-one children were classed as conservers of interior volume.

The correlation between the scores on the two interior volume items is .88. The correlation between the combined scores for the conservation of area and the combined scores for the conservation of interior volume is .73. The correlations between the two area items and the subtotal for interior volume are .53 and .79 respectively.

When a one-way analysis of variance was applied to the sample grouped according to performance on the Barns Test, there were no significant differences between these groups on any of the other variables.

B. Perceptual Efficiency

The ability to discriminate, to remember and to represent were investigated via the visual, auditory and tactual modalities. The means and standard deviations of the scores on the nine tests of perceptual efficiency are shown in Table VII, for the total sample.

Correlations between the total sample's scores on the tests of perceptual efficiency are shown in Table VIII.

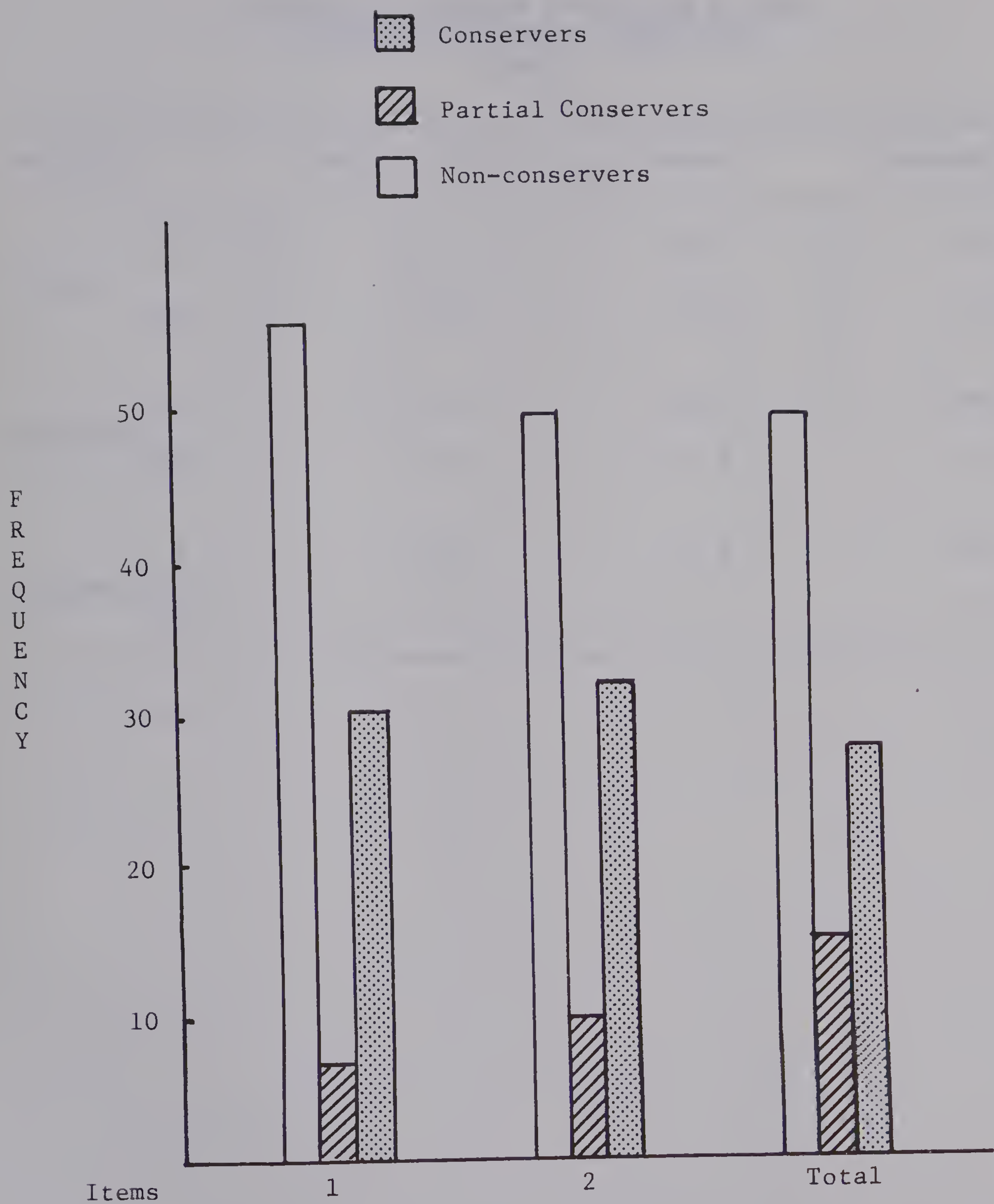


FIGURE 12

DISTRIBUTION OF SCORES ON THE TESTS OF CONSERVATION OF INTERIOR VOLUME

TABLE VII

MEANS AND STANDARD DEVIATIONS OF THE
TESTS OF PERCEPTUAL EFFICIENCY
(N=90)

Modality		Discrimination	Memory	Representation
Visual	$\overline{X}$	28.7	24.4	27.0
	SD	8.8	3.1	3.8
Auditory	$\overline{X}$	7.2	43.8	10.5
	SD	4.0	7.6	4.5
Tactual	$\overline{X}$	4.8	5.2	27.2
	SD	1.7	1.7	5.3

TABLE VIII

CORRELATIONS BETWEEN THE TESTS OF PERCEPTUAL EFFICIENCY FOR THE TOTAL SAMPLE (N=90)

		Visual			Auditory			Tactual		
		Discrimination	Memory	Representation	Discrimination	Memory	Closure	Discrimination	Memory	Representation
VISUAL	Discrimination				.26**	.48**	.42**			
	Memory									
	Representation					.24*				.25*
AUDITORY	Discrimination					.42**	.26**		.23*	
	Memory						.55**		.26**	.25*
	Closure							.31**		.23*
TACTUAL	Discrimination								.30**	
	Memory									.26**
	Representation									

**Significant at the .01 level
*Significant at the .05 level

In order to make comparisons, a subject whose score was equivalent to the highest five scores on each test was categorized as a top performer. Similarly, a subject whose score was equivalent to the lowest five scores on each test was categorized as a bottom performer on that test. Altogether there were forty-eight children classed as top performers and forty-two classed as bottom performers, on these tests of perceptual efficiency.

1. Visual Modality

The three tests used to measure discrimination, memory and representation in the visual modality were

a. Huelsman-Word Discrimination Test Form B - Alta

b. Test of Visual Memory

c. Test of Visual Representation.

a. Visual Discrimination

The distribution of scores on the Huelsman-Word Discrimination Test Form B-Alta ranged from three to fifty-six and is shown in Figure 13. All but one of the lowest seven scores belonged to children who spoke a language other than English at home. The other boy heard three non-English languages spoken at home though he could not converse in any of them. All of the children spoke normal English without an obvious foreign accent.

The girl with the top score of fifty-six was a top performer on the auditory discrimination test.

b. Visual Memory

The range and distribution of the scores for the Test of

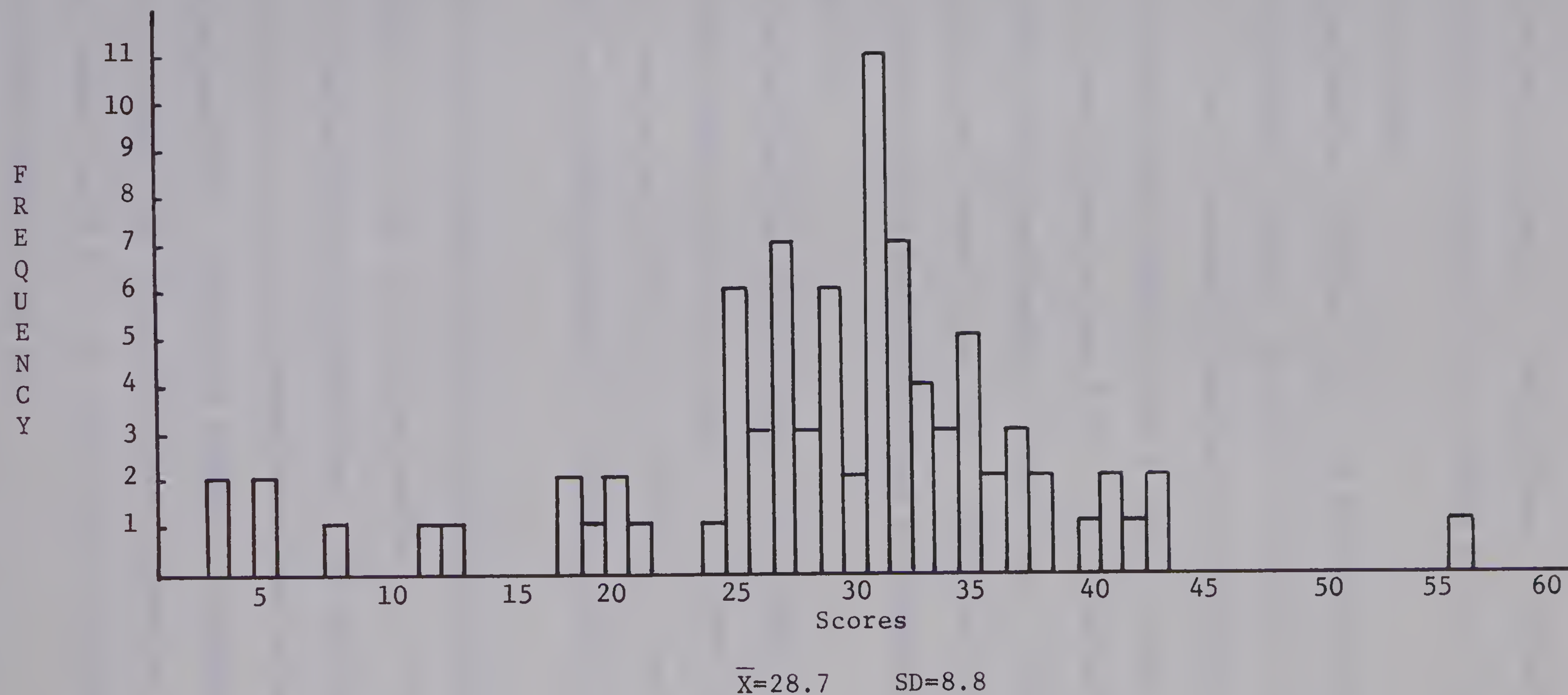


FIGURE 13

FREQUENCY DISTRIBUTION OF VISUAL DISCIMINATION SCORES (BASED ON THE HUELSMAN-WORD DISCIMINATION TEST FORM B-ALTA) FOR THE TOTAL SAMPLE (N=90)

Visual Memory are shown in Figure 14. The children with the six lowest scores belonged to the high (N = 1), middle (N = 3), and low (N = 2) mathematics groups.

The high mathematics achiever was a top performer in auditory discrimination and auditory closure. He read at a Grade 3.7 level (Michael, age 8.5). One of the middle achievers in mathematics was a top performer in visual discrimination and auditory closure. She read at a Grade 5.4 level (Giselle, age 8.11). The other middle mathematics achievers were bottom performers on other tests. For one, it was tactual discrimination (Darrell, age 8.4). The other was so categorized in auditory closure and both tactual discrimination and memory (Mike, age 8.4).

The two low mathematics achievers who were classed as bottom performers on visual memory were Willie (age 8.6) and Debra (age 9.8). This was the only test on which Willie was classed as a top or bottom performer. Debra was a bottom performer on auditory memory and auditory closure.

The three most successful children on the visual memory test, all girls, belonged to the high (N = 1), and middle (N = 2) mathematics groups. The high mathematics achiever was a top performer in auditory closure as well as tactual discrimination and tactual representation (Helga, age 8.4). The middle achievers were both girls. Visual memory was the only test on which one was classed as a top or bottom performer (Maria, age 8.8). The other was a top performer on auditory discrimination (Cheryl, age 9.2).

The correlation between the scores for visual memory and

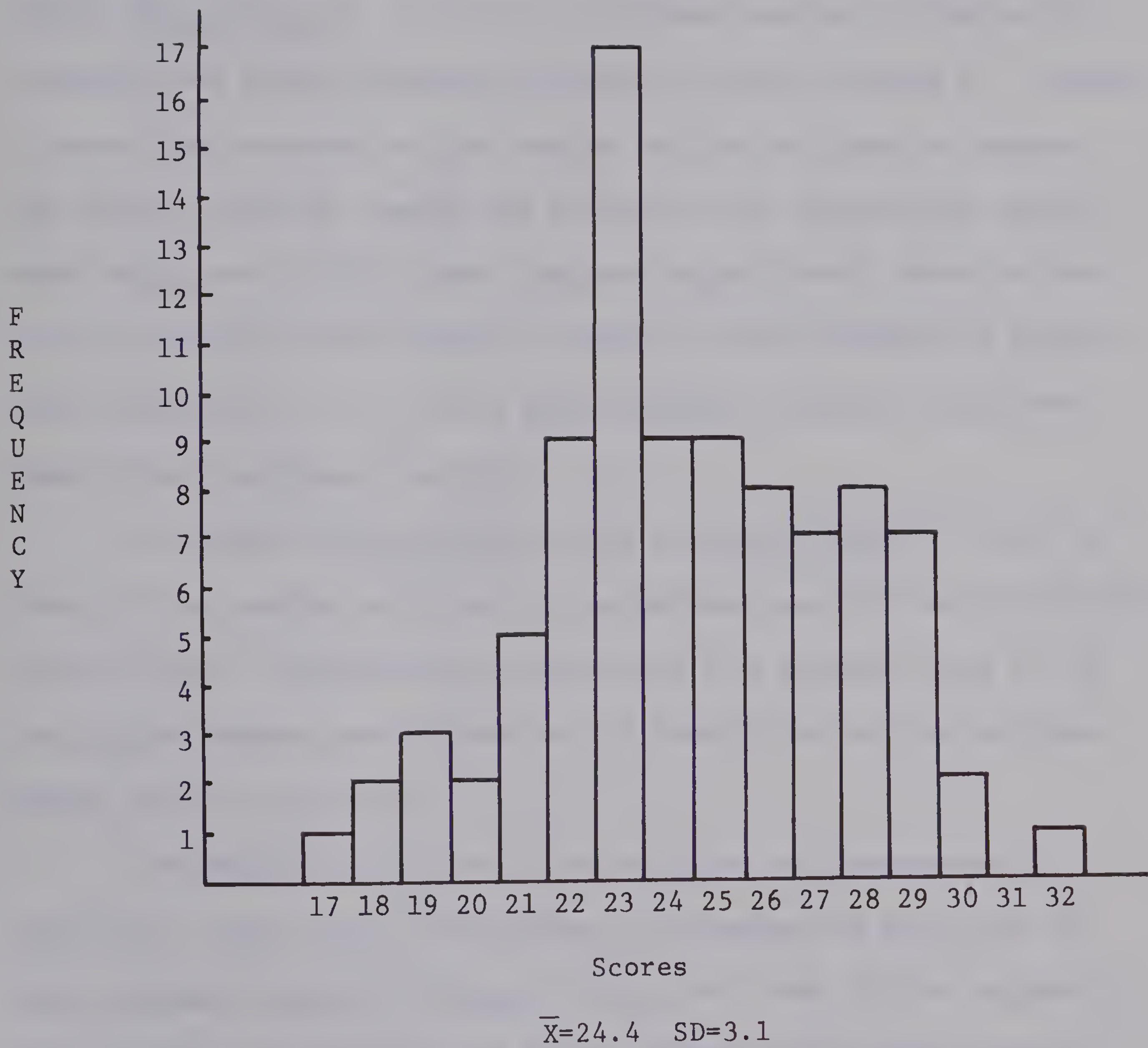


FIGURE 14

FREQUENCY DISTRIBUTION OF VISUAL MEMORY SCORES
FOR TOTAL DISTRIBUTION (N=90)

visual discrimination was not significant ($p \geq .05$).

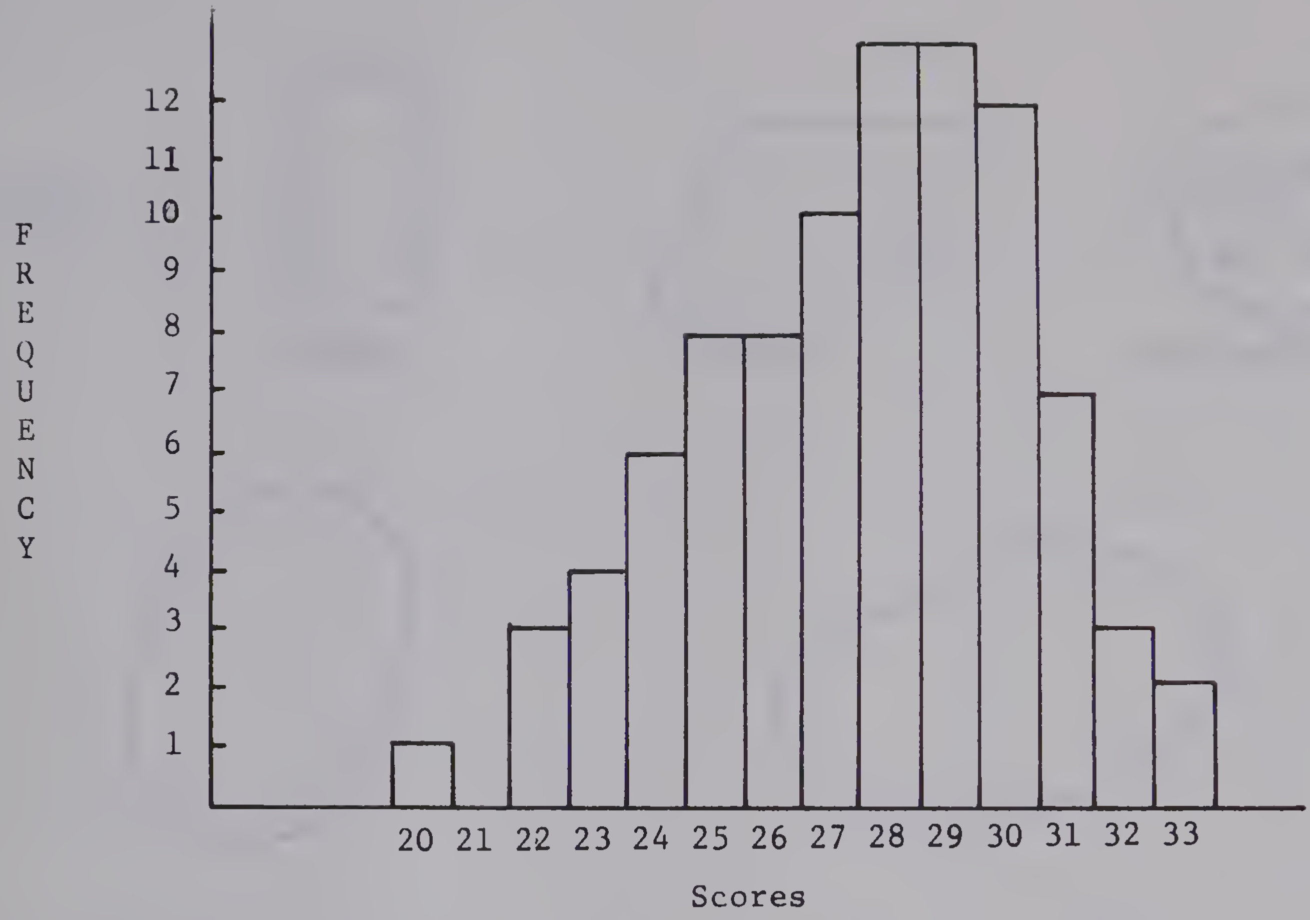
c. Visual Representation

The range and distribution of the scores on the Test of Visual Representation, in which the children copied by drawing five euclidean and three topological forms are shown in Figure 15. Figure 16 shows the breakdown of the results for the two types of shapes. The scores clustered towards the ceiling in the topological section where eighty-one of the ninety subjects scored seven, eight or nine. This is consistent with Piaget's contention that topological properties come earlier in a child's understanding of spatial relations than do the euclidean properties.

The subject who had the lowest score of twenty was rated at Grade 5.0 on reading and Grade 4.4 on mathematics, with an intelligence level of 121. She gained five points out of a possible nine on the topological shapes, and fifteen out of twenty-five on the euclidean shapes (Elaine, age 8.8).

Drawings made by four of the children are reproduced in Figure 17. Gail's (age, 9.8) oblong and rhombus are both oval in shape and seem closest to Piaget's Stage IB. Most of his subjects at this stage are just four years of age (Piaget and Inhelder, 1967, pp. 56-68). Her score on the test was twenty-one.

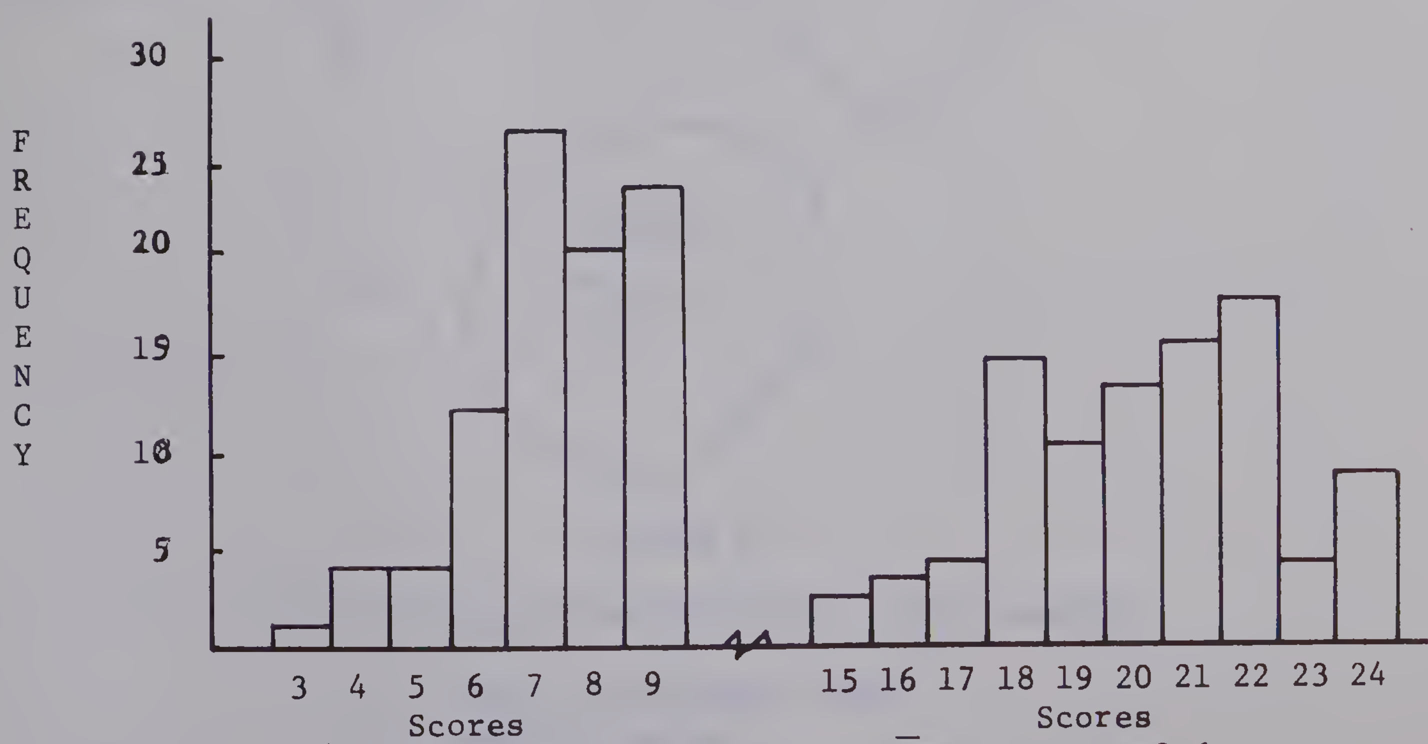
The drawings of the rhombus by two of the boys cannot be matched to one of Piaget's samples. However, it would appear that they could be categorized at Stage IB. One boy, with a perfect score on the topological shapes, gained twenty-seven points



$\overline{X}$=27.0 SD=3.8

FIGURE 15

FREQUENCY DISTRIBUTION OF VISUAL REPRESENTATION SCORES FOR THE TOTAL SAMPLE (N=90)



$\overline{X}$=7.3 SD=1.4
TOPOLOGICAL

$\overline{X}$=19.7 SD=3.6
EUCLIDEAN

FIGURE 16

DISTRIBUTION OF THE SCORES ON THE TOPOLOGICAL AND EUCLIDEAN SECTIONS OF THE TEST OF VISUAL REPRESENTATION FOR THE TOTAL SAMPLE (N=90)

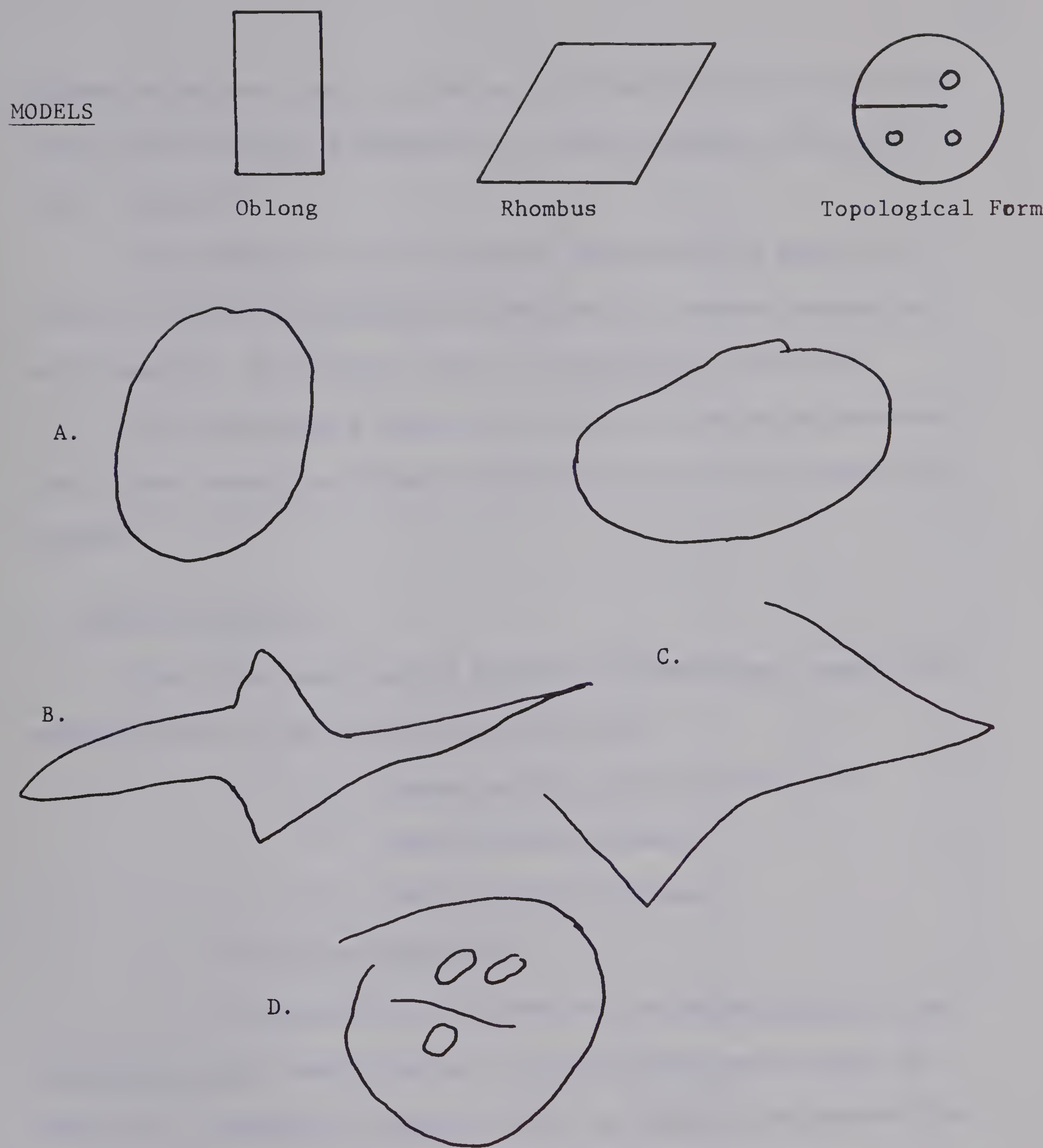


FIGURE 17

TEST OF VISUAL REPRESENTATION - SOME DRAWINGS

A. Oblong and Rhombus (Gail)
B. Rhombus (Arthur)
C. Rhombus (Chris)
D. Topological Form (Willie)

altogether (Arthur, age 9.2), while the other boy with five out of nine on the topological shapes had a total of twenty-five points (Chris, age 8.11).

The drawing of the topological form by Willie (age 8.6) failed on the three topological properties of closure, separation and proximity. He scored a total of twenty-two for the test.

The correlations between the scores for visual representation, visual memory and visual discrimination were not significant ($p \geq .05$).

2. Auditory Modality

The three tests used to measure discrimination, memory and representation in the auditory modality were

a. Wepman Auditory Discrimination Test

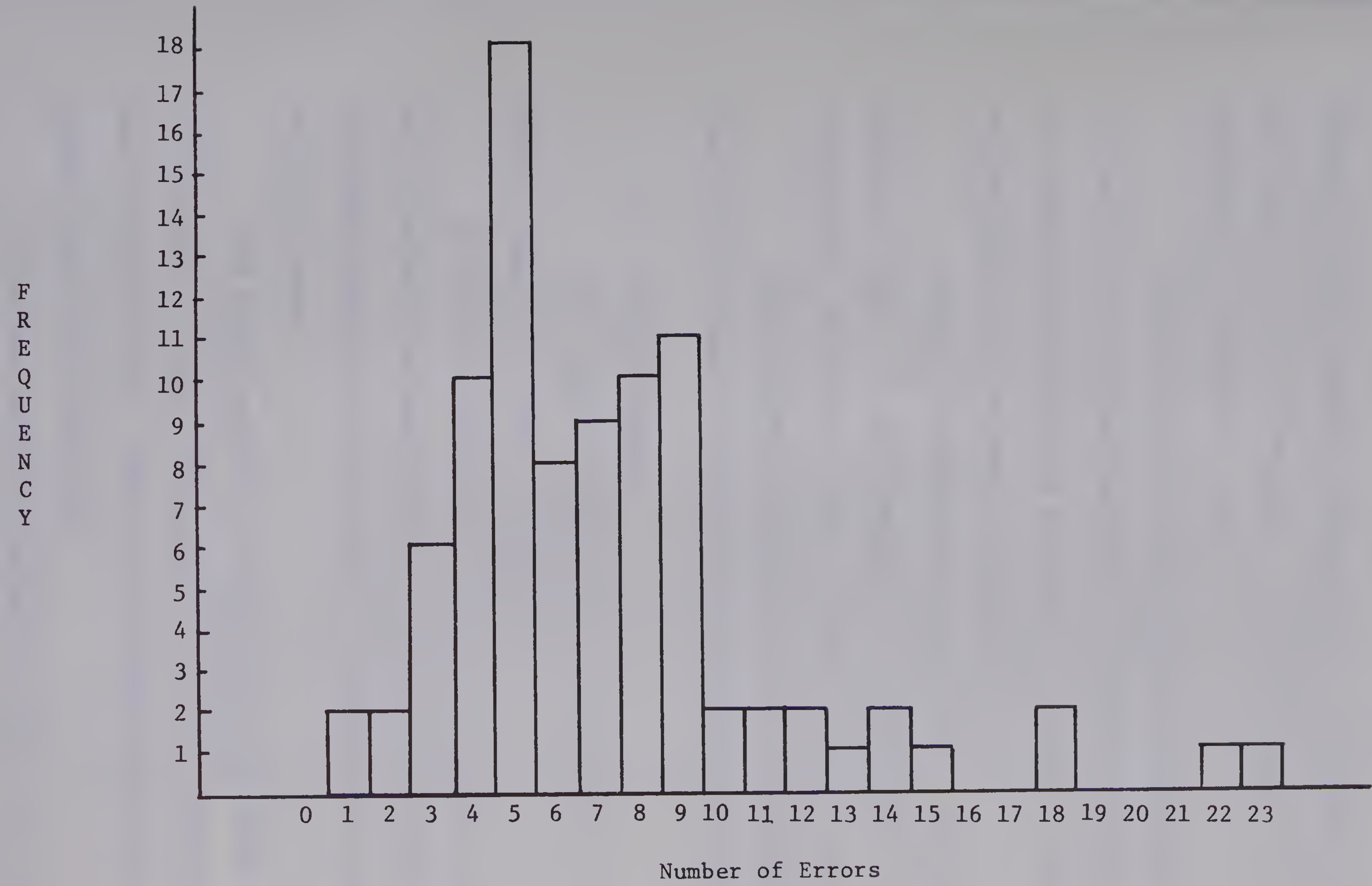
b. Test of Auditory Memory

c. Test of Auditory Closure.

a. Auditory Discrimination.

The distribution of errors on the Wepman Auditory Discrimination Test ranged from one to twenty-three and is shown in Figure 18. According to Wepman (1958), an adequate performance for the child who is eight years or older is three or less errors. Eighty of the ninety subjects did not meet this criterion.

The four least adequate performers belonged to the low mathematics achievement group. The girl with twenty-three errors had difficulty in the training session, but seemed to understand when the question was asked, "Would you draw the same picture?" Her



$\overline{X}$=7.2 SD=4.0

FIGURE 18

FREQUENCY DISTRIBUTION OF AUDITORY DISCRIMINATION ERRORS (BASED ON THE WEPMAN AUDITORY DISCRIMINATION TEST) FOR THE TOTAL SAMPLE (N=90)

response pattern indicated perseveration. Her reading score put her at a Grade 1.6 level. She was a top performer on the Test of Tactual Representation (Rosemarie, age 8.5).

The two boys each with eighteen errors read at the Grade 1.6 level also. One was generally a low performer (Ezia, age 8.10). The other was a conserver of both area and interior volume and was a top performer in visual memory (Barry, age 8.3).

The boy with twenty-two errors was a bottom performer in auditory closure. He was classed as a top performer in tactual discrimination. He read at a Grade 3.2 level (Robert, age 9.7).

The correlation between the scores on the discrimination tests in the visual and auditory modalities was .26 ($p<.01$).

b. Auditory Memory.

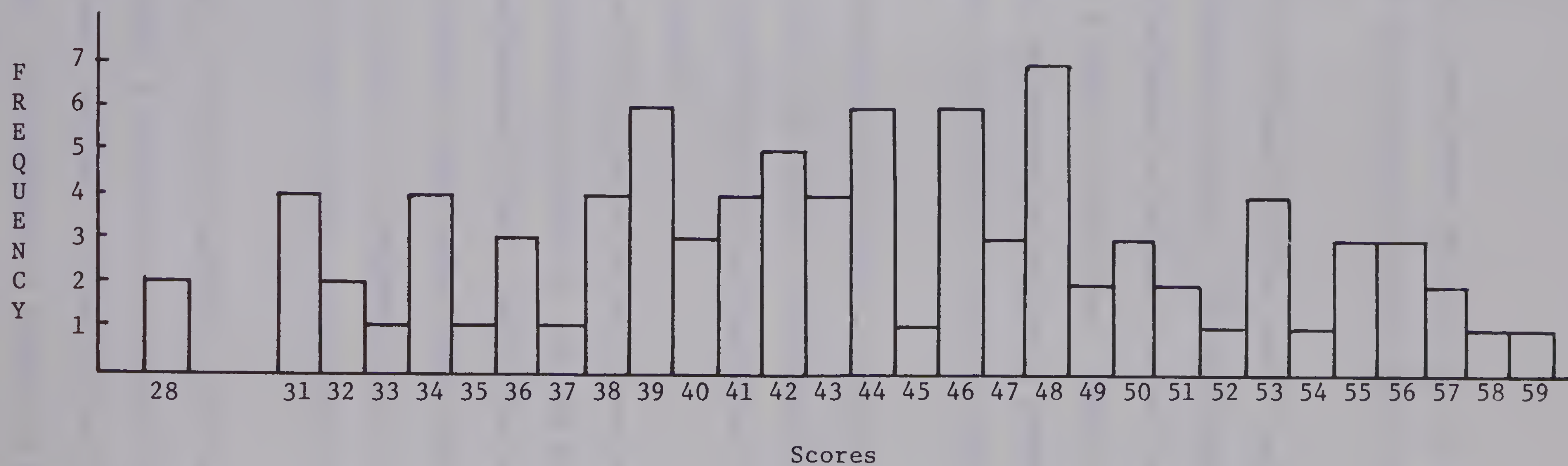
The range and distribution of the scores on the Test of Auditory Memory are shown in Figure 19.

It will be recalled that this test consists of sentences which gradually increase in length starting with a single word. The fourteenth sentence in both length and complexity of grammatical structure reads:

"The market was full of people buying all kinds of things."

A child able to repeat the sentences correctly to this item would earn forty-two points. Thirty-five children or 38.9 per cent of the sample scored below this level.

In the high achieving mathematics group, five subjects scored below forty-eight, and of these one was below forty-two. This boy



$\overline{X}$=43.8 SD=7.6

FIGURE 19

FREQUENCY DISTRIBUTION OF AUDITORY MEMORY SCORES FOR TOTAL SAMPLE (N=90)

with thirty-four points scored above the mean for the sample on the three visual tests but below the mean for the three auditory tests (Roman, age 8.8).

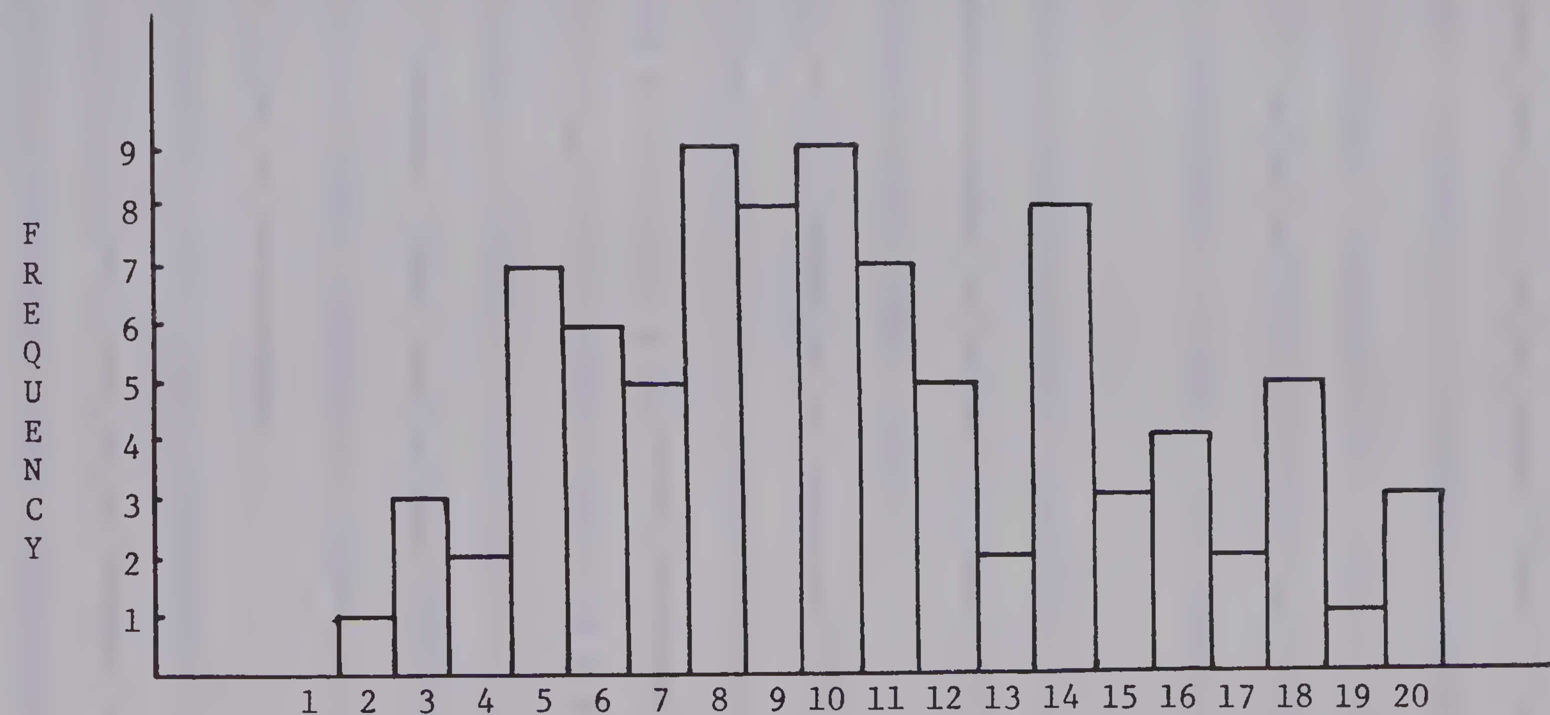
The nine bottom performers on this test of auditory memory belonged to the low mathematics group. With one exception, the highest score of a low mathematics achiever was forty-six. The exception obtained the top score of fifty-nine. She was classed as a top performer in visual representation and was a conserver of both area and interior volume. She read at a Grade 5.1 level (Therese, age 8.5).

The correlation between the scores on auditory memory and auditory discrimination was .42 ($p<.01$). The correlation between auditory memory and visual memory was not significant ($p\geq.05$).

c. Auditory Closure.

The range and distribution of the scores on the Test of Auditory Closure are shown in Figure 20. Of the six lowest scores, five were in the low mathematics group. Some of the only girl's responses followed a pattern. Sheet became shoe, kitty - kick me, string - ring, potato - eight oh. A response such as moon for policeman does not fit into a category easily. She gave no nonsense words (Debbie, age 9.8).

Similarly, one of the boys attempted to give back English words. He seemed to retain a sound from a previous item which he incorporated into the next response. Six words after wastebasket, microscope became basket. The response of eskimo for pencil does



Scores
$\overline{X}$=10.5 SD=4.5
FIGURE 20

FREQUENCY DISTRIBUTION OF AUDITORY CLOSURE
SCORES FOR TOTAL SAMPLE (N=90)

not fit a simple category (Roberto, age 8.10).

Ten of another boy's responses, all wrong, finished with a t or at sound and were mostly nonsense words (Robert, age 9.7). Two boys did not respond according to any pattern. Policeman became omit, cafeteria - package, transportation - dragon (Andre, age 8.5). The responses of the other boy fell into the same category. Microscope became hero, caterpillar - speak, hippopotamus - snake (Dale, age 8.4).

The remaining bottom performer in auditory closure had an above average problem-solving score which raised him into the middle mathematics achievement group (Mike, age 8.4).

The normative data assembled by Knights (1966) provide only gross measures for comparison with the present study. For eight-year-olds the mean score is 10.6 with a standard deviation of 4.7. The mean age on the children in the present study was 8.8 years. The mean of the total sample on auditory closure was 10.5 with a standard deviation of 4.5. However, there are extreme differences between the means of the low and high mathematics groups, a finding which will be discussed later in this chapter.

The correlation of .55 ($p<.01$) between auditory closure and auditory memory was the highest correlation between any two of the nine tests of perceptual efficiency. The correlation between the scores for auditory closure and visual representation was not significant ($p\geq.05$).

3. Tactual Modality

The three tests used to measure discrimination, memory and representation in the tactual modality were

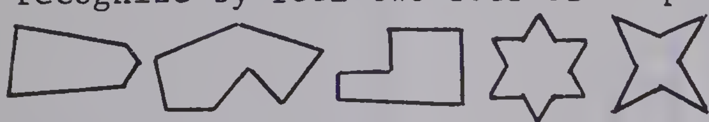
a. Test of Tactual Discrimination

b. Test of Tactual Memory

c. Test of Tactual Representation.

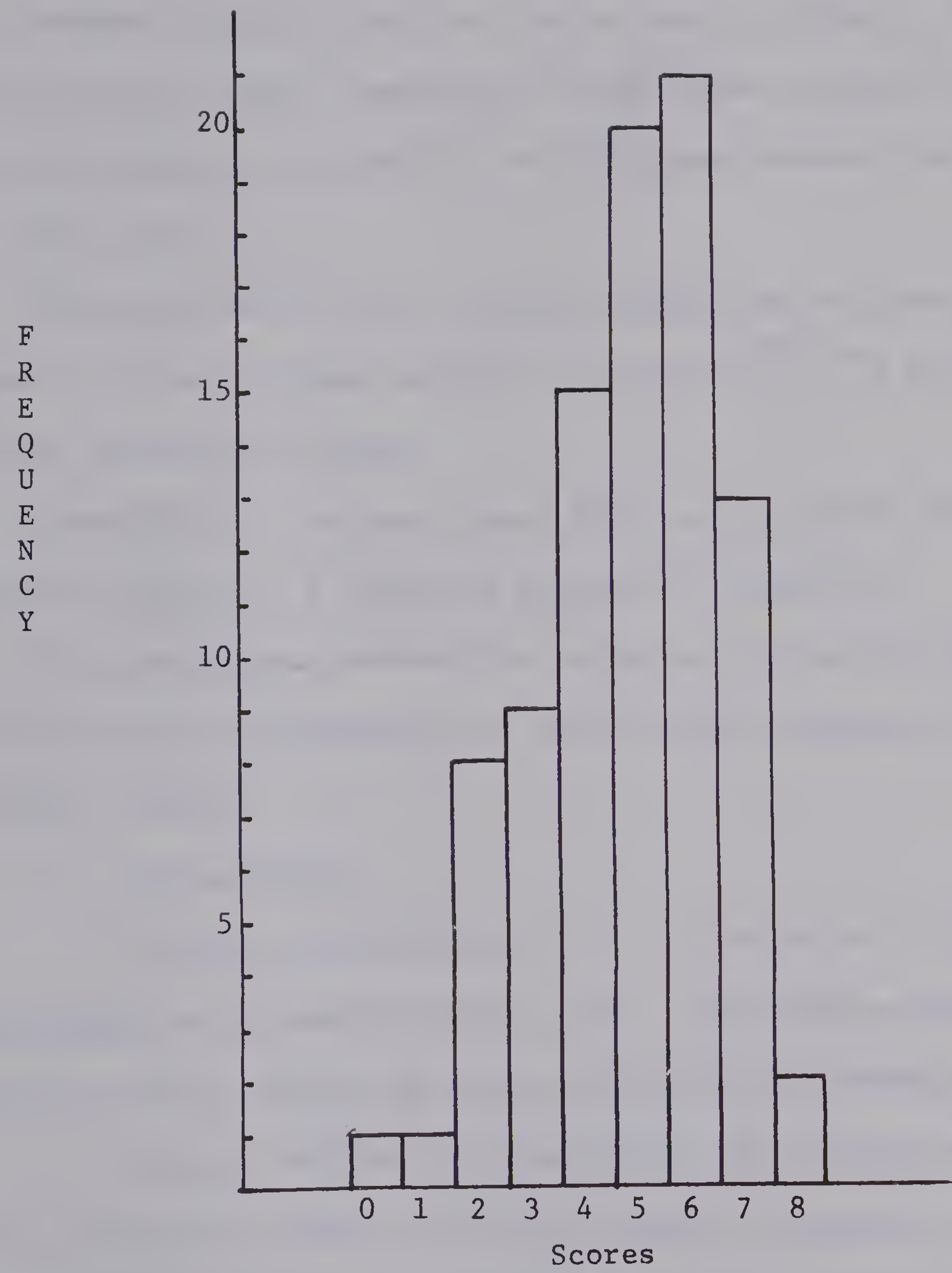
a. Tactual Discrimination.

The range and distribution of scores on the Test of Tactual Discrimination are shown in Figure 21.

It will be recalled that the children were required to recognize by feel two sets of shapes ( ). The shapes in the first set contained convex angles, while the other four shapes had at least one concave angle.

The two boys with perfect performance were both in the high mathematics group. One was two years above the mean of the sample in reading and was a conserver of both area and interior volume (Zenon, age 8.3). The other had the second top score in the entire sample in visual discrimination. An average reader and a partial conserver, he was a bottom performer in tactual representation (Dale, age 9.2).

The boy who scored zero was also in the high mathematics group. He was a bottom performer on the auditory discrimination test. During the tactual discrimination test, the only time he did not select the shape  in the second set was when it would have been correct (Gregory, age 8.3).

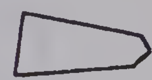


$\overline{X}$=4.8 SD=1.7

FIGURE 21

FREQUENCY DISTRIBUTION OF TACTUAL DISCRIMINATION SCORES FOR TOTAL SAMPLE (N=90)

The eight children who gave two correct responses were evenly divided between the middle and the low mathematics groups. No pattern appears to occur in their responses, though three children interchanged the two star shapes. One boy felt no difference between the two star shapes (Mike, age 8.4).

There does seem to be a pattern amongst the children who made one error. Of the thirteen subjects, six chose  for  while four confused the stars.

Page (1959), from those study this test is taken, does not present his results in a form that allows for comparison.

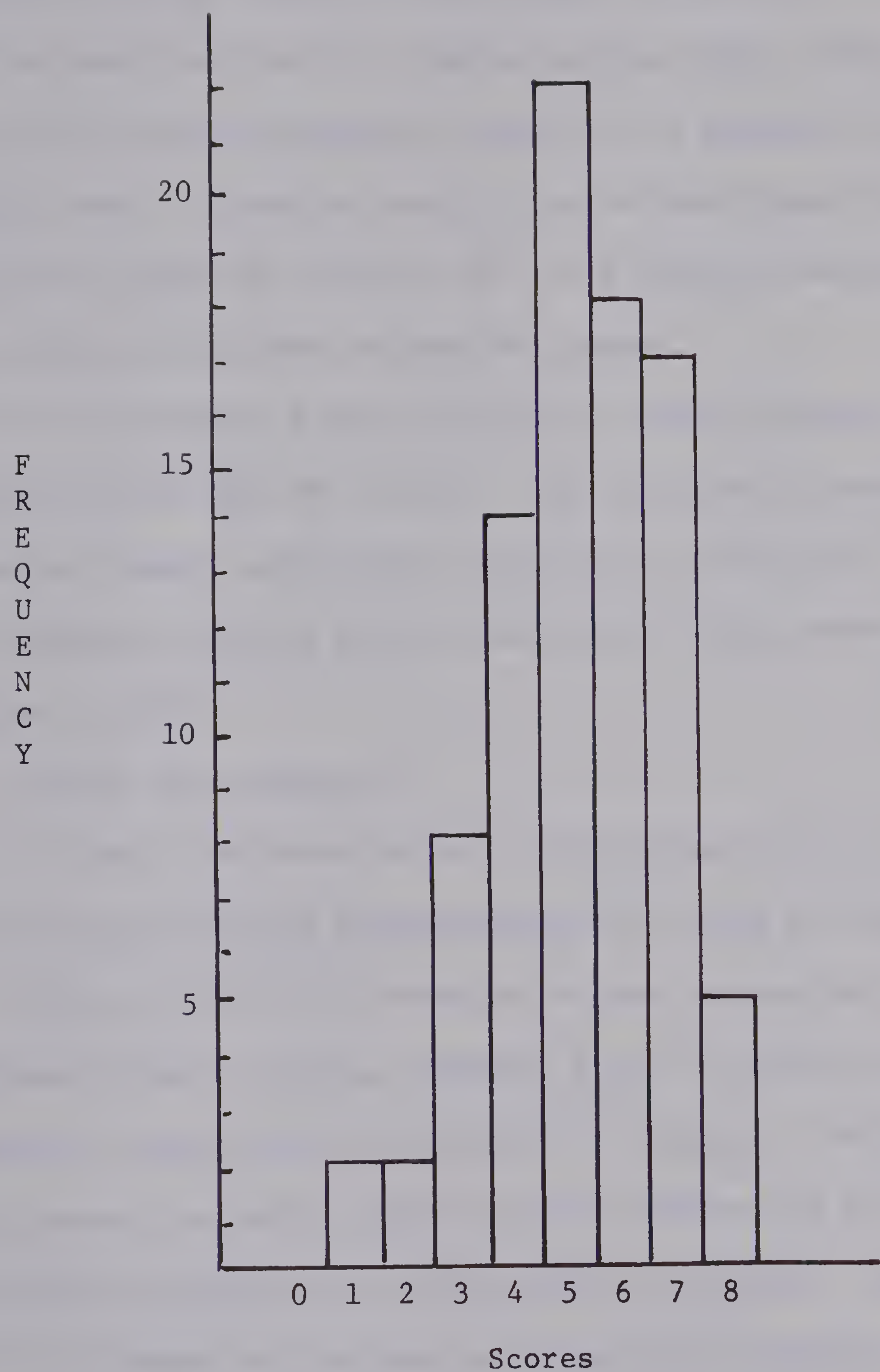
The correlations between the scores on tactual discrimination and either visual discrimination or auditory discrimination were not significant ($p \geq .05$).

b. Tactual Memory

The range and distribution of the scores on the <u>Test of Tactual Memory</u> are presented in Figure 22. The stimulus shapes were identical to those used in the Test of Tactual Discrimination.

The four children who had perfect performances belonged to the high mathematics group. The boy who made no mistakes on the discrimination form of these tasks scored two when memory was involved (Zenon, age 8.3).

The thirteen lowest scorers were spread evenly over the three mathematics groups with the extra one belonging to the high group. One boy chose the four-pointed star for each of the shapes in the second set. He varied his response on the convex shapes which



$\overline{X}=5.2$ SD=1.7

FIGURE 22

DISTRIBUTION OF SCORES ON TACTUAL MEMORY
FOR TOTAL SAMPLE (N=90)

were alternated with the concave forms (Joseph, age 8.10).

The mean for the total sample on this memory test was 5.2, compared to 4.8 on the discrimination tasks with a standard deviation of 1.7 in each case. Altogether twenty-five children made more mistakes on this test than the previous one. Six children were three or more worse on this second time through the shapes.

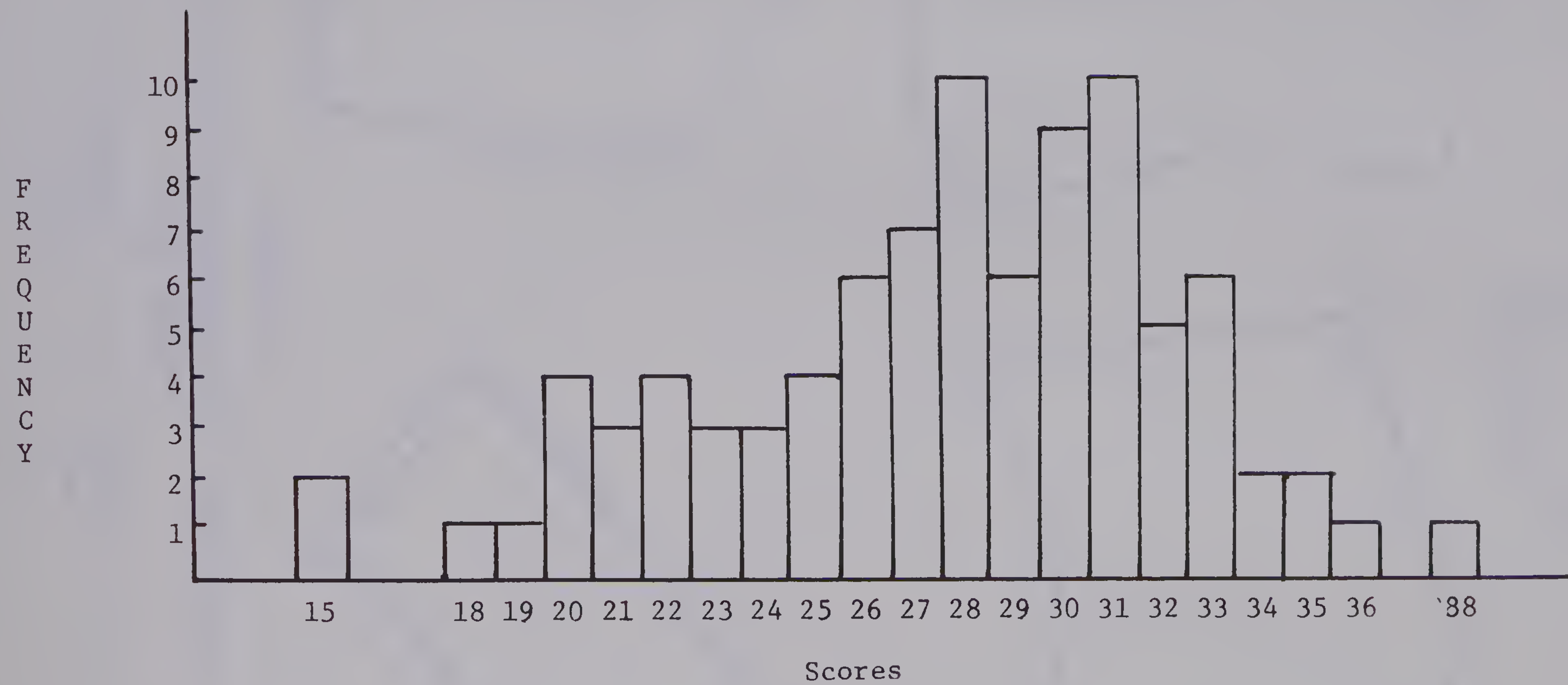
The correlation between scores for tactual memory and tactual discrimination was .30 ($p<.01$). The correlation between scores on tactual memory and auditory memory was .26 ($p<.01$). The correlation between scores on tactual memory and visual memory was not significant ($p \geq .05$).

c. Tactual Representation

The range and distribution of the scores for the total sample on the <u>Test of Tactual Representation</u> are shown in Figure 23.

Each child felt and attempted to draw unseen eight euclidean shapes (square, oblong, rhombus, kite, two similar right-angled triangles, equilateral and isosceles triangles). The two shapes which caused the most trouble were the rhombus and the kite, which six children confused with an equilateral triangle. Samples of drawings of the rhombus and the kite are presented in Figures 24 and 25. Altogether eleven children, of whom eight were girls, scored full points for their drawings of the rhombus, while the kite was drawn successfully by two pupils, both girls.

The only child to draw both these shapes correctly belonged to the low mathematics group and was reading at a Grade 1.9



$\overline{X}$=27.2 SD=5.3

FIGURE 23

FREQUENCY DISTRIBUTION OF TACTUAL REPRESENTATION
SCORES FOR TOTAL SAMPLE (N=90)

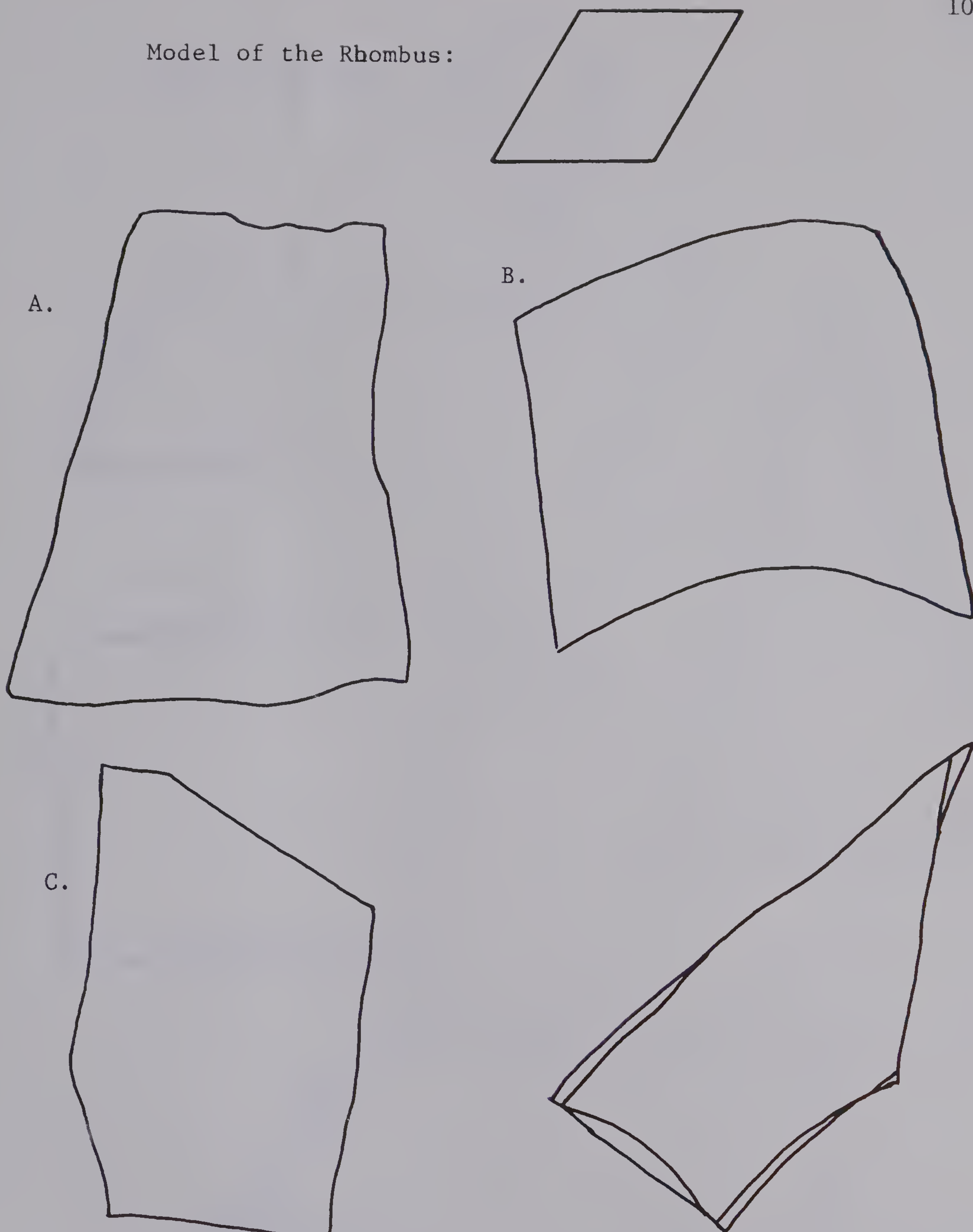


FIGURE 24

TEST OF TACTUAL REPRESENTATION - SOME DRAWINGS OF THE RHOMBUS

A. Perry C. Vinko
B. Carmela D. Mike

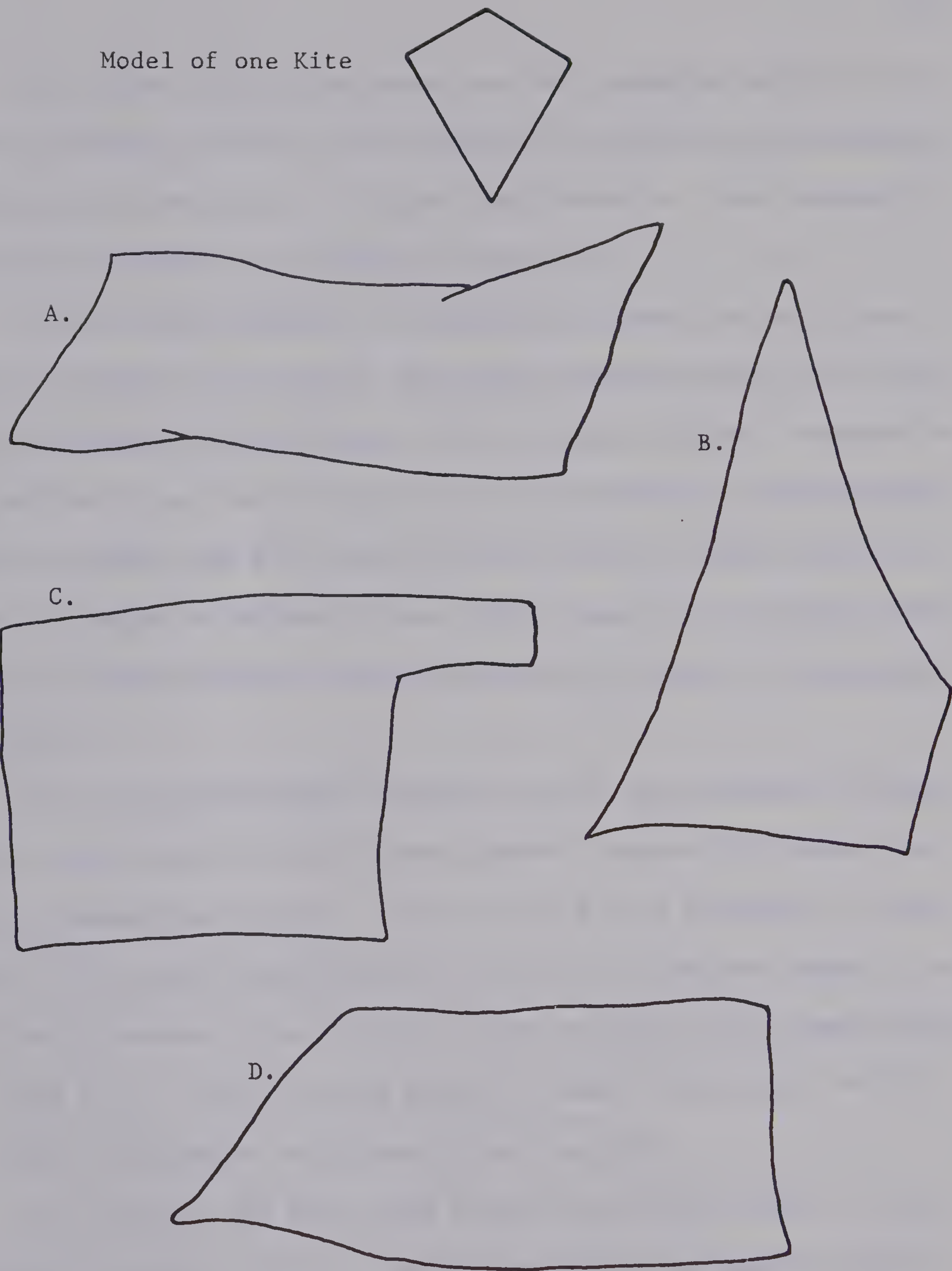


FIGURE 25

TEST OF TACTUAL REPRESENTATION - SOME DRAWINGS
OF THE KITE

A. Clara C. Ezia
B. Frank D. Natalie

level. She scored thirty-four points out of a possible forty for the whole test (Helen, age 9.0). The other girl to draw the kite successfully was a top performer in auditory discrimination. She belonged to the middle mathematics group (Barbara, age 8.11).

Of the eleven subjects who gained full points for their drawing of the rhombus, five were in the high mathematics group, with three each in the middle and low groups. The low group children consisted of Helen and two boys. One of these was a top performer in visual representation (Franco, age 8.7), while the other had the lowest score of the entire sample on auditory closure (Dale, age 8.4). The boys earned thirty and thirty-two points respectively for this test of tactual representation.

The six top performers included four of the successful rhombus drawers. The other two earned almost perfect marks on all shapes except the rhombus and the kite. The girl was bottom performer in both auditory and tactual discrimination as well as in auditory memory. Her visual discrimination score was well below the mean of the sample (Rosemarie, age 8.5). The boy scored above the mean of the sample on all nine tests of perceptual efficiency (Marc, age 8.6).

Two boys and two girls made up the four lowest scores. A boy and a girl belonged to each of the low and middle mathematics groups. One girl with a score of fifteen drew the four different triangles as if they were all equilateral (Gail, age 9.8). Also with a score of fifteen, the other girl failed to gain points for any shape on three criteria, namely presence of straight lines, accuracy of straight

lines, accuracy of angles and accuracy of lines (Clara, age 8.6). No pattern is apparent in one boy's drawings which gained eighteen points (Perry, age 9.2).

The other boy's drawings, which scored nineteen points, are shown in Figure 26. This boy was a bottom performer in visual discrimination. He read well over a year below the expectations of his chronological age (Arthur, age 9.2).

The correlation between the scores for tactual representation and tactual memory was .26 ($p<.01$). The correlation between the scores for tactual representation and tactual discrimination was not significant ($p\geq.05$). The correlation between scores for tactual representation and auditory closure, the equivalent test in the auditory modality, was .23 ($p<.05$). The correlation between the scores for tactual representation and visual representation was .25 ($p<.05$).

4. Comparison of Performances

There were forty-eight children who were classed as top performers on one or more of the tests of perceptual efficiency. Forty-two children were classed as bottom performers.

These top and bottom performers can be categorized further. The categories are

i. those who were top performers only;

ii. those who were bottom performers only;

iii. those who were classed as a top performer on at least one test as well as a bottom performer on one or more tests;

iv. those who were not classed as a top or bottom performer on any of the nine tests of perceptual efficiency.

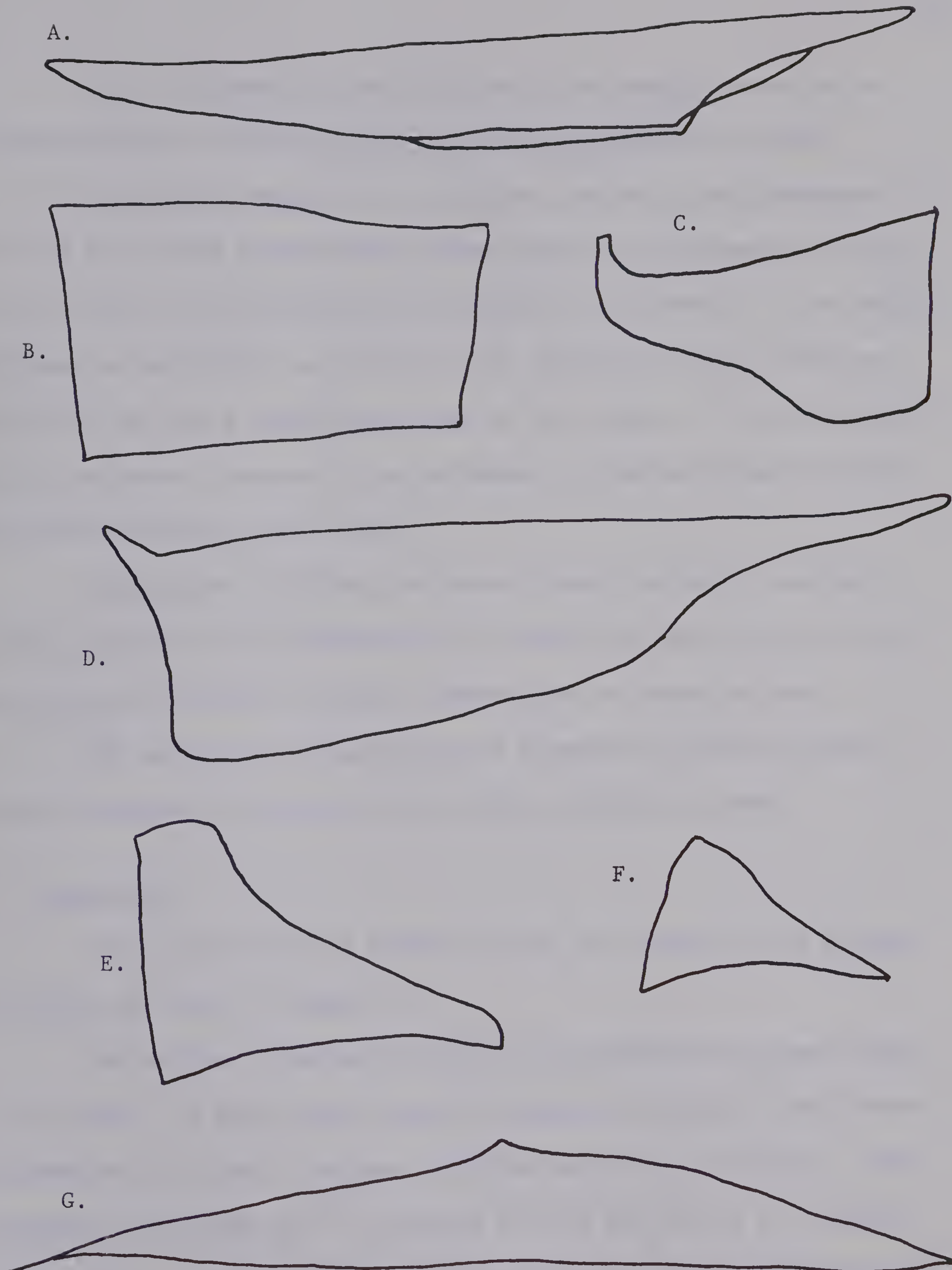


FIGURE 26

TEST OF TACTUAL REPRESENTATION - ONE BOY'S DRAWINGS

A. Equilateral triangle
B. Square
C. Kite
D. Rhombus
E. Large right-angled triangle
F. Small right-angled triangle
G. Isosceles triangle

Table IX shows the distribution of the sample according to these categories over the three mathematics achievement groups.

Of the high mathematics achievers, two were top performers in six and four tests respectively, three were top performers in three tests, while three were bottom performers in two tests. Of the middle mathematics achievers, no one was a top performer on more than two tests and one was a bottom performer on four tests. Of the low mathematics achievers, one was a top performer in two tests and two were bottom performers in four tests.

The sixteen children, who were classed as both a top and a bottom performer, had combinations of either one and one, or two and one, with two subjects having a combination of three and one.

The majority of those who were classed as neither top nor bottom performers belonged to the middle mathematics group.

C. Estimation

The distribution of scores for the four items of the Estimation Test is shown in Figure 27.

One subject from each of the three mathematics groups scored on one item. In each case it was the numerical quantity item involving marbles in a jar. Thirteen children scored on two items. They belonged to the high (N = 1), middle (N = 5) and low (N = 7) mathematics groups. Twelve of these did not score on the area-number item which required the subject to co-ordinate estimation of area with numerical quantity. Nine did not score on the area item.

Two boys, both in the low mathematics group, earned the maximum

TABLE IX

DISTRIBUTION OF THE SAMPLE AS TOP AND BOTTOM PERFORMERS OVER THE THREE MATHEMATICS GROUPS (N=90)

	Mathematics Groups			Total Sample
	High	Middle	Low	
Top Performers Only	16	9	7	32
Bottom Performers Only	7	6	13	26
Both Top and Bottom Performers	5	6	5	16
Neither Top nor Bottom Performers	2	9	5	16
Total	30	30	30	90

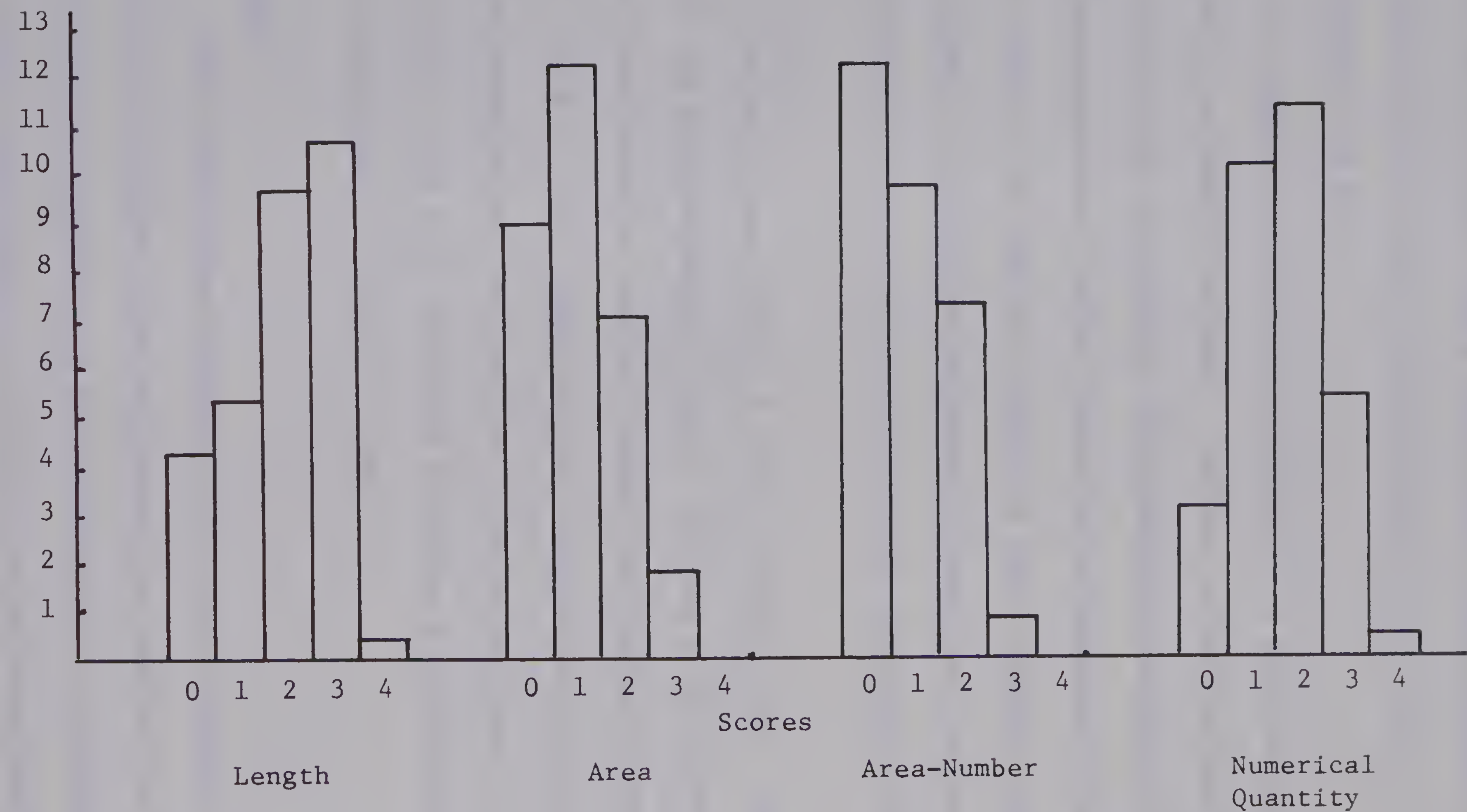


FIGURE 27

FREQUENCY DISTRIBUTION OF SCORES ON ESTIMATION TASKS
FOR TOTAL SAMPLE (N=90)

points on any one test. One's success was on the numerical quantity item (Mick, age 9.1), while the other gained full points in the length item (Richard, age 9.1).

The highest total score, obtained by a girl and a boy, was ten points out of a possible sixteen. The boy, a high mathematics achiever, was a top performer in auditory discrimination, auditory closure and visual representation. He was classed as a conserver of both area and interior volume. He read at a Grade 5.8 level (Dermot, age 9.1). The girl, a member of the middle mathematics group, was neither a top nor a bottom performer in any test of perceptual efficiency (Carmela, age 9.1).

Three girls each scored a total of nine points. They all belonged to the high mathematics group. One was a top performer in auditory discrimination, auditory memory and tactual representation (Gladys, age 8.7). The other two were both bottom performers in visual representation. One was also a bottom performer in tactual memory (Elaine, age 8.8), while the other was a bottom performer in tactual representation (Angelina, age 9.1).

D. Summary

In the descriptive analysis of the data, attention has been focused on the children whose performance put them at the extremities of the frequency distributions of the scores for the various tests.

Of the ninety subjects, eighty-four performed better than the mean of the sample on at least two of the abilities tested under conservation, perceptual efficiency and estimation. Of the remaining

six children, three belonged to one class, and two belonged to another. One of the six failed to meet the mean on any one test.

III. RESULTS OF TESTING THE HYPOTHESES

Hypothesis One

There is no significant difference between the scores obtained by the high, middle and low achievers in mathematics in tests of

a. conservation

b. perceptual efficiency

c. estimation

Results

a. Conservation of Area and Interior Volume

The means and standard deviations of the scores on the four conservation items for the three mathematics groups are presented in Table X. In each case the high mathematics group scored the best on the conservation items. The low mathematics group scored the lowest on the conservation items.

When a one-way analysis of variance (ANOV 15) was applied to the data as shown in Table XI, there were no significant differences ($p \geq .05$) between the three mathematics groups on the two items of area conservation. The high and low mathematics groups differed significantly ($p < .05$) on the two interior volume items. Differences involving the middle mathematics group did not reach a level of significance.

Because of strong correlations between mathematics achievement and intelligence (.71, $p < .001$) and between intelligence and reading

TABLE X

MEAN SCORES FOR CONSERVATION OF AREA AND INTERIOR VOLUME FOR THE HIGH, MIDDLE AND LOW MATHEMATICS GROUPS (N=90)

	Mathematics Groups							
	High Group (N=30)		Middle Group (N=30)		Low Group (N=30)		Total Group (N=90)	
	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD
Area								
Item 1	1.03	1.00	1.00	1.01	.87	.97	.96	.99
Item 2	1.06	.91	.86	.90	.57	.86	.83	.90
Interior Volume								
Item 1	1.06	.98	.57	.86	.50	.86	.71	.93
Item 2	1.13	.97	.73	.90	.50	.82	.79	.93

TABLE XI

SUMMARY OF ANALYSIS OF VARIANCE ON CONSERVATION OF AREA AND INTERIOR VOLUME OVER HIGH, MIDDLE AND LOW MATHEMATICS GROUPS (N=90)

Test	Source of Variance and Sums of Squares		Mean Squares		df		F
	Among Means of Total Scores	Within Scores	Among Means of Total Scores	Within Scores	Among Means of Total Scores	Within Scores	
Conservation of Area							
Item 1	.47	.86	.23	.99	2	87	.23
Item 2	.38	.69	1.90	.79	2	87	2.41
Conservation of Interior Volume							
Item 1	.57	.70	2.88	.81	2	87	3.54*
Item 2	.61	.70	3.08	.81	2	87	3.78*

*Significant at the .05 level

(.80, $p < .001$) for the total sample, a one-way analysis of covariance, controlling for differences in intelligence, was applied to the data. There were no significant differences ($p \geq .05$) for conservation of either area or interior volume.

b. Perceptual Efficiency

The means and standard deviations of the scores on the nine tests of perceptual efficiency for the three mathematics groups are shown in Table XII. The high mathematics group had the highest means, and the low mathematics group had the lowest means on all the tests, except for tactual representation, when the low group surpassed the middle group, and for visual representation when the middle group surpassed the high group.

When a one-way analysis of variance was applied to the data, as shown in Table XIII, there were no significant differences ($p \geq .05$) between the three mathematics groups on the scores for visual memory and representation, and tactual discrimination, memory and representation. Significant differences ($p < .05$) existed on the scores for visual discrimination, auditory discrimination, memory and closure.

On the Sheffé comparison of means, there were significant differences ($p < .01$) between the high and low mathematics groups on visual discrimination and auditory discrimination, memory and closure. The middle mathematics group was significantly different from the high group on auditory closure ($p < .01$) and from the low group on auditory discrimination ($p < .05$), while on auditory memory all three groups were significantly different ($p < .05$).

TABLE XII

MEAN SCORES AND STANDARD DEVIATIONS FOR THE TESTS OF PERCEPTUAL EFFICIENCY FOR THE HIGH, MIDDLE AND LOW MATHEMATICS GROUPS

	High Group (N=30)		Middle Group (N=30)		Low Group (N=30)		Total Group (N=90)	
	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD
Visual Discrimination	33.6	7.5	28.6	9.1	23.9	7.1	28.7	8.8
Visual Memory	25.1	2.9	24.5	3.5	23.8	2.8	24.4	3.1
Visual Representation	26.8	4.0	28.0	1.2	26.4	2.3	27.0	3.8
Auditory Discrimination	6.0	3.3	6.3	2.1	9.3	5.3	7.2	4.0
Auditory Memory	49.0	6.2	44.1	6.0	38.2	6.6	43.8	7.6
Auditory Closure	14.2	3.8	9.5	3.7	7.8	3.4	10.5	4.5
Tactual Discrimination	5.4	1.6	4.6	1.8	4.5	1.6	4.8	1.7
Tactual Memory	5.8	1.7	5.0	1.5	4.8	1.6	5.2	1.7
Tactual Representation	28.2	4.7	26.0	6.5	27.3	4.5	27.2	5.3

TABLE XIII

SUMMARY OF ANALYSIS OF VARIANCE FOR THE TESTS OF PERPETUAL EFFICIENCY OVER THE HIGH, MIDDLE AND LOW MATHEMATICS GROUPS (N=90)

	Test	Source of Variance and Sums of Squares		Mean Squares		df		F
		Among Means of Total Scores	Within Scores	Among Means of Total Scores	Within Scores	Among Means of Total Scores	Within Scores	
VISUAL	Discrimination	1402.00	5541.31	701.00	63.69	2	87	11.01*
	Memory	2282.20	8275.04	11.41	9.51	2	87	1.20
	Representation	41.12	1292.75	20.56	14.86	2	87	1.38
AUDITORY	Discrimination	202.22	1257.57	101.11	14.45	2	87	6.99*
	Memory	1744.81	3433.31	872.41	39.46	2	87	22.11*
	Closure	657.42	1153.07	328.71	13.25	2	87	24.80*
TACTUAL	Discrimination	13.49	239.63	6.74	2.75	2	87	2.45
	Memory	16.42	232.73	8.21	2.68	2	87	3.07
	Representation	71.37	2458.43	35.69	28.26	2	87	1.26

*Significant at the .01 level

When a one-way analysis of covariance, with differences in intelligence controlled, was applied to the data, there were no significant differences ($p \geq .05$) between the three mathematics groups on the scores for visual memory, and tactual discrimination, memory and representation. Significant differences ($p < .05$) existed on the scores for visual discrimination and representation, and auditory discrimination, memory and closure.

The raw means and the adjusted means for the scores of the nine tests of perceptual efficiency,with differences in intelligence controlled, are shown in Table XIX in Appendix A.

c. Estimation

The means and standard deviations of the scores on the estimation tasks for the three mathematics groups are shown in Table XIV.

TABLE XIV

MEAN SCORES FOR ESTIMATION OVER THE HIGH, MIDDLE AND LOW MATHEMATICS GROUPS (N=90)

Mathematics Groups							
High Group (N=30)		Middle Group (N=30)		Low Group (N=30)		Total Group (N=90)	
$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
6.5	2.0	5.2	1.9	4.6	1.9	5.4	2.1

A one-way analysis of variance was applied to the total estimation scores (Table XV), followed by a Sheffé comparison of means. Significant differences ($p<.05$) existed between the high mathematics group and both the middle and low groups. The difference between the middle and low groups did not reach significance ($p\geq.05$).

When a one-way analysis of covariance, with differences in intelligence controlled, was applied to the data, there was no significant difference among the three mathematics groups on estimation.

d. Other analyses

In order to check whether sex was a critical variable, a two-analysis of covariance (ANCV 20), was applied to the data. An interaction between sex intelligence and mathematics was found. The girls had the higher intelligence in the high and middle mathematics groups; the boys had the higher intelligence scores in the low mathematics group. On the scores for conservation, perceptual efficiency and estimation, differences were not significant ($p\geq.05$) when differences in intelligence were accounted for.

In order to check if conservation was a critical variable, a one-way analysis of variance was applied to the data, with the subjects being grouped as conservers, partial conservers and non-conservers. There were no significant differences ($p\geq.05$) amongst these groups on the tests of perceptual efficiency and estimation.

Conclusion

On the basis of these results, Hypothesis One was accepted for conservation of area, visual memory and representation, and tactual

TABLE XV

SUMMARY OF ANALYSIS OF VARIANCE ON THE TEST OF ESTIMATION OVER THE HIGH, MIDDLE AND LOW MATHEMATICS GROUPS

Test	Source of Variance and Sums of Squares		Mean Squares		df		F
	Among Means of Total Scores	Within Scores	Among Means of Total Scores	Within Scores	Among Means of Total Scores	Within Scores	
Estimation	54.95	335.27	27.48	3.85	2	87	7.13*

*Significant at the .01 level

discrimination, memory and representation. On these variables there were no significant differences among the three mathematics achievement groups.

Hypothesis One was rejected for conservation of interior volume, visual discrimination, auditory discrimination, memory and closure, and estimation. On these variables, significant differences existed among the three mathematics achievement groups.

When differences in intelligence were controlled, significant differences existed among the three mathematics achievement groups on visual discrimination and representation. and auditory discrimination, memory and closure.

Hypothesis Two

There is no significant correlation between scores obtained on tests of

a. conservation of area and interior volume

b. perceptual efficiency

c. estimation

and mathematics achievement.

Results

The correlation between conservation of area and interior volume was .73 ($p<.001$). The correlation of .30 between conservation of interior volume and mathematics achievement was significant ($p<.01$).

The correlations between the nine tests of perceptual

efficiency for the total sample were shown in Table VIII. Auditory memory correlated significantly with six of the other eight variables. The two variables with which auditory memory did not correlate significantly were visual memory and tactual discrimination. The three auditory abilities correlated significantly with each other. Tactual memory correlated significantly with the other tactual abilities and auditory discrimination and memory. Visual discrimination correlated significantly with the three auditory tests. Visual memory did not correlate significantly with any of the other variables ($p<.05$).

The correlations between the perceptual efficiency scores and the mathematics scores, shown in Table XVI, were significant for the three auditory abilities, discrimination in the visual and tactual modalities, and tactual memory. Correlations between the tests of perceptual efficiency and intelligence and reading are also presented in Table XVI.

Conservation of area correlated significantly with auditory memory ($r = .21$, $p \geq .05$), while conservation of interior volume correlated significantly with visual representation ($r = .26$, $p<.01$), auditory discrimination ($r = .23$, $p<.05$), auditory memory ($r = .29$, $p<.01$), auditory closure ($r = .26$, $p<.01$) and estimation ($r = .23$, $p<.05$).

The scores for estimation correlated significantly with those for mathematics ($r = .39$, $p<.01$), and conservation of interior volume ($r = .23$, $p<.05$). Significant correlations existed with

TABLE XVI

CORRELATIONS BETWEEN MATHEMATICS, INTELLIGENCE AND READING SCORES, AND THE TESTS OF PERCEPTUAL EFFICIENCY (N=90)

		Mathematics	Intelligence	Reading
VISUAL	Discrimination	.44**	.47**	.49**
	Memory			
	Representation		.30**	
AUDITORY	Discrimination	.36**	.26**	.35**
	Memory	.60**	.53**	.62**
	Closure	.60**	.49**	.55**
TACTUAL	Discrimination	.23*	.23*	.24*
	Memory	.28**	.23*	
	Representation		.20*	.23*

**Significant at .01 level of probability
*Significant at .05 level of probability

discrimination in the visual modality ($r = .29$, $p<.01$) and in the tactual modality ($r = .24$, $p<.05$); with auditory memory ($r = .35$, $p<.01$) and closure ($r = .26$, $p<.01$).

Table XX showing the correlations between conservation of area and interior volume, the nine tests of perceptual efficiency, estimation and mathematics achievement for the total sample as well as the high, middle and low mathematics achievement groups, is to be found in Appendix B.

Conclusion

There is a possible total of seventy-eight correlations between the thirteen variables categorized under

a. conservation
b. perceptual efficiency
c. estimation, and
d. mathematics achievement.

Of these, thirty-four were significant.

Auditory memory correlated significantly ($p<.05$) with ten of the other twelve variables. Auditory discrimination and auditory closure were involved in six and eight significant correlations respectively. The three tactual abilities and visual discrimination appeared in either four or five significant correlations. Conservation of interior volume was involved in seven significant correlations, as was estimation in six. Visual memory did not correlate significantly with any other variable ($p\geq.05$).

The four variables which did not correlate significantly

with mathematics achievement were conservation of area, visual memory and representation, and tactual representation ($p \geq .05$).

Although a considerable number of significant correlations did appear, the hypothesis must be accepted with regard to the total set of variables.

IV. ADDITIONAL FINDINGS

Two additional analyses were applied:

a. stepwise regression

b. factor analysis

Stepwise regression was chosen in order to discover which variables best predicted mathematics achievement. Furthermore, this type of analysis could indicate from a different point of view the qualitative as well as the quantitative differences between the high, middle and low mathematics groups.

The data was subjected to factor analysis to find out if commonalities between the variables existed. The number of factors having eigenvalues greater than or equal to one was six. Two further analyses limited to two and five factors were applied. The five factor analysis was selected for examination because it seemed to group the variables into principal components with most clarity.

a. stepwise regression

The stepwise regression analysis for the ninety Grade 3 subjects indicated that intelligence was the best predictor of mathematics achievement. It accounted for 51.9 per cent. Auditory closure increased

this percentage to 60.6. The inclusion of age lifted the proportion to 64.3 per cent, while auditory memory raised the predictability to 66.71 per cent ($p<.05$). These results are shown in Table XVII.

When the high and middle mathematics groups were taken together, auditory closure was the best predictor of success in mathematics achievement, accounting for 28.6 per cent. The inclusion of sex raised the proportion to 38.7 per cent, while estimation brought it up to 49.5 per cent ($p<.05$). When intelligence scores were not included in the data being analysed, the results remained the same for auditory closure and sex. However, visual discrimination then entered to raise the prediction to 43.5 per cent ($p<.05$).

When the middle and low mathematics groups were taken together, intelligence was the best predictor of mathematics achievement, accounting for 30.6 per cent. The next variable entering was age which raised the percentage to 40.2. Visual memory then brought the proportion predicted to 44.4 per cent ($p<.05$). When intelligence scores were not included in the data being analysed, auditory memory accounted for 18.6 per cent ($p<.05$) of the mathematics achievement score.

b. factor analysis

Table XVIII presents the principal components from the factorial treatment of the seventeen test variables included. The correlation matrix for these variables is shown in Table XXI (Appendix C). The minimum loading for inclusion in a principal component was .40, a limit accepted by Kerlinger (1964, p. 654). All the variables had sufficient

TABLE XVII

STEPWISE PREDICTION OF MATHEMATICS ACHIEVEMENT
FOR THE TOTAL SAMPLE (N=90)

Step	Predictors Included in Equation	Fv	Pv	Percentage of Variance Accounted For
1	Intelligence	95.15	.000	51.95
2	Intelligence Auditory Closure	19.09	.000	60.60
3	Intelligence Auditory Closure Age	8.95	.003	64.31
4	Intelligence Auditory Closure Age Auditory Memory	6.07	.016	66.69

TABLE XVIII

PRINCIPAL COMPONENTS ARISING FROM THE FACTOR ANALYSIS OF THE TEST VARIABLES

	COMMONALITIES	1	2	3	4	5
1.	0.751	0.794	0.185	0.276	0.071	-0.074
2.	0.724	0.762	0.175	0.126	0.310	-0.022
3.	0.756	0.797	0.132	0.168	0.232	-0.148
4.	0.585	0.745	-0.149	0.053	0.063	-0.033
5.	0.696	0.674	0.177	0.145	0.295	-0.319
6.	0.576	0.684	0.155	0.272	0.084	0.048
7.	0.284	0.439	0.185	0.235	-0.041	-0.002
8.	0.635	0.044	0.762	-0.228	-0.006	-0.006
9.	0.842	0.123	0.904	0.082	-0.024	0.054
10.	0.833	0.069	0.870	0.170	0.186	-0.086
11.	0.886	0.083	0.901	0.210	0.131	-0.077
12.	0.580	0.135	0.014	0.725	-0.056	0.059
13.	0.661	0.007	0.024	0.779	0.227	-0.054
14.	0.667	0.045	0.230	-0.147	0.769	-0.004
15.	0.681	0.051	-0.162	0.269	0.762	0.009
16.	0.775	0.191	0.100	0.243	0.064	0.815
17.	0.561	-0.288	-0.143	-0.351	-0.059	0.576

TABLE XVIII (continued)

Variables

1. Mathematics Achievement
2. Intelligence
3. Reading
4. Visual Discrimination
5. Auditory Memory
6. Auditory Closure
7. Estimation
8. Barns Test - Conservation of Area
9. Transformed Triangles Test - Conservation of Area
10. Rearranged Cube Test - Conservation of Interior Volume
11. Islands Test - Conservation of Interior Volume
12. Tactual Discrimination
13. Tactual Memory
14. Visual Representation
15. Tactual Representation
16. Auditory Discrimination
17. Visual Memory

loadings to be included in one, and only one, of the five factors.

Factor I contained mathematics, intelligence and reading together with visual discrimination, auditory memory and closure, all with loadings above .66, and estimation with a loading of .43. Except for estimation, the tests used to measure these variables had an emphasis on language comprehension, whether in the written or spoken form. Although the Huelsman Word-Discrimination Test Form B-Alta (visual discrimination) did not demand word knowledge, the items involved real words rather than meaningless sets of letters.

Factor II, a conservation factor, contained the four tasks measuring conservation of area and interior volume.

Factor III was tactual in nature. The tests of tactual discrimination and tactual memory used the same stimulus shapes though the tasks differed.

The tests making up Factor IV required the children to draw shapes, presented visually in one case and in the other tactually. Thus this factor could be described as Representation.

Factor V was a combination of visual memory and auditory discrimination. There seems to be no obvious commonality to explain the relationship. These two variables did not correlate significantly when subjected to the Pearson Product Moment correlation.

V. SUMMARY

This chapter contains an analysis of the various instruments used in this study, and the results of testing two hypotheses which

were associated with the major purposes of the study outlined earlier.

The purposes of this study were to investigate in one group of children, conservation of area and interior volume, perceptual efficiency and estimation; and from there to determine the relationship, if any, of these variables to mathematics achievement. It was found on all tests except those of visual and tactual representation that the high mathematics group had the highest means, and the low mathematics group had the lowest means. The differences between the groups were significant at the .05 level for conservation of interior volume, visual discrimination, auditory discrimination, memory and closure, and estimation. When differences in intelligence were accounted for, significant differences existed among the three mathematics groups on visual discrimination and representation, and auditory discrimination, memory and closure.

Significant correlations were found between mathematics achievement and most of the other variables, as well as amongst these variables. The most prominent of these variables being involved in significant correlations was auditory memory, followed by auditory closure, conservation of interior volume, auditory discrimination and estimation.

Auditory closure and auditory memory were also prominent in the stepwise regression analysis, whether the sample was taken as a whole or categorized according to high, middle and low mathematics achievement.

The factor analysis resulted in five factors which could be described as i. Language Comprehension, ii. Conservation, iii. Tactual, iv. Representation, with a fifth one a combination of visual memory and auditory discrimination.

Implications arising from these findings will be discussed in the next chapter.

CHAPTER V

SUMMARY, DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

I. SUMMARY OF THE INVESTIGATION

The present study was designed primarily to investigate in one group of Grade 3 children a number of variables that seemed related to mathematics ability, and then to determine the relationship, if any, of these variables to mathematics achievement. The variables selected were:

i. conservation of area and interior volume;

ii. perceptual efficiency in the visual, auditory and tactual modalities, specifically in relation to their functions in discrimination, memory and representation tasks, and

iii. estimation of length, area and numerical quantity.

Spatial concepts provided the unifying theme.

Sample

The sample of ninety Grade three children was chosen from eight classrooms within the Edmonton Separate School System, on the basis of performance on the mathematics section of the Canadian Test of Basic Skills. Thirty subjects were randomly selected to form each of three groups, high, middle and low mathematics achievers. The mathematics achievement scores of the sample ranged from Grade 1.6 to Grade 5.5, with a mean of Grade 3.6. The ages of the children ranged from ninety-six to one hundred and twenty-two months, with a mean of

105.4 months. The intelligence levels, as measured by the *California Test of Mental Maturity*, ranged from sixty-three to one hundred and twenty-seven, with a mean of 104.4. It was assumed that this sample was representative of urban children of that grade level.

Instruments

A battery of tests was administered to each subject, either in group situations or individually. The four tasks used to measure conservation of area and interior volume were Piagetian in nature. Nine tests were selected to measure

a) visual discrimination, memory and representation;

b) auditory discrimination, memory and closure; and

c) tactual discrimination, memory and representation.

The stimulus material for five of these perceptual tests featured geometric shapes. The scores for estimation were drawn from items testing estimation of length, area and numerical quantity. Standardized tests were used to measure mathematics achievement, reading and intelligence.

Conclusions

A summary of the findings will be presented on the basis of testing the hypotheses, which in broad terms were concerned with the significant differences and relationships among a number of variables considered related to mathematics achievement. No significant differences were found amongst the high, middle and low mathematics groups on conservation of area, visual memory and representation and the

three tactual abilities. Conversely, significant differences were found on the other six variables, namely, conservation of interior volume, visual discrimination, the three auditory abilities and estimation. In each case the differences between the high and low groups was significant. The high and middle groups were significantly different on auditory memory and closure and estimation, while the low and middle groups were significantly different on auditory discrimination and memory.

When differences in intelligence were controlled, the three auditory abilities, along with visual discrimination and representation, were the variables which showed significant differences among the three mathematics groups.

On the basis of the above results pertaining to significant differences, Hypothesis One was not rejected for conservation of area, visual memory and representation and the three tactual abilities, but was rejected for conservation of interior volume, visual discrimination, auditory discrimination, memory and closure, and estimation.

When conservation of area and interior volume, the nine tests of perceptual efficiency, estimation and mathematics achievement were categorized as thirteen variables, significant correlations were found between mathematics achievement and eight of the other variables. Amongst the variables grouped under perceptual efficiency, the three auditory abilities were prominent. Auditory memory correlated significantly with all the variables except visual memory and tactual discrimination. Auditory closure and discrimination each contributed

to eight and six significant correlations respectively; while conservation of interior volume and estimation did so in seven and six cases respectively.

The number of significant correlations among the thirteen variables was thirty-four. Mathematics achievement and the three auditory abilities were involved in twenty-nine of these correlations.

The number of possible correlations amongst thirteen variables is seventy-eight. Therefore, on the basis of the above findings, Hypothesis Two was not rejected.

When additional analyses were applied, the auditory abilities continued to be prominent. In the stepwise regression auditory memory and auditory closure stood out, whether the sample was taken as a whole or categorized according to high, middle and low mathematics achievement.

A factor analysis of the data produced five independent factors which could be described as Language Comprehension, Conservation, Tactual, Representation, with a fifth one a combination of visual memory and auditory discrimination.

II. DISCUSSION OF SOME OF THE FINDINGS

a. Results of Some of the Findings

Conservation of area was measured by the Barns Test and the Transformed Triangles Test, both being derived from Piagetian investigations. Of the forty-two children categorized as conservers on the Barns Tests, twenty-seven seemed to be thinking about the numerical

equivalence of the barns in each field, rather than the idea of occupied regions, a rationalization expressed by Piaget's conservers. It may be that the Barns Test is in fact testing number conservation rather than area conservation. The administration of a classical test of number conservation, in this study, may have been useful in verifying or refuting this assertion.

Piaget states that conservation of area and interior volume occur in that sequence. If the Transformed Triangles Test is taken as an indication of the conservation of area, then Piaget's claim is generally substantiated by the results of this study. Of the sixty-nine subjects not classified as conservers of both area and interior volume, only five deviated from the Piagetian sequence.

Of the twenty-one children who were conservers of both area and interior volume, ten belonged to the high, five to the middle and six to the low mathematics groups. However, the distribution of the thirty-two non-conservers in the three mathematics groups were nine, eleven and twelve respectively. Although the high mathematics achievers tended to be more successful on the tests of conservation, the ability to conserve area and interior volume as measured here did not appear to be a critical condition for success on the mathematics achievement test. The authors of the Canadian Test of Basic Skills claim that the test measures knowledge of mathematical concepts. This claim does not appear to agree with Piaget's (1952) statement that "conservation is a necessary condition for all mathematical understanding" (p. 4). When referring to the Canadian Test of Basic

Skills, Birch (Buros, 1972) maintains that "for the present this is probably as useful an instrument as exists" (p. 6). Whatever the case there are three alternatives. Either this test is not testing mathematical understanding, the measures used in this study are not really measures of conservation, or Piaget is wrong.

Each of the nine tests of perceptual efficiency can be classified in two ways, according to either its modality or to one of the three phases of discrimination, memory and representation. None of the three visual tests correlated significantly with each other, while all of the auditory tests did so. Tactual memory correlated significantly with the other two tactual abilities. There was a significant correlation between discrimination in the visual and auditory modalities, but not in the tactual modality. Memory in the auditory and tactual modalities correlated significantly. It will be recalled that visual memory did not correlate significantly with any other ability. Tactual representation correlated significantly with representation in the visual and auditory modalities.

When the perceptual efficiencies are categorized according to modality only and phase only, there are eighteen possible correlations. Of this number, nine were significant. Six involved an auditory ability while five involved memory in either the auditory or tactual modalities.

In currently published programs for early elementary school pupils, any planned experiences designed to improve a child's perceptual efficiency are usually directed at the visual modality, in connection with the development of pre-reading skills. The results of this

study would indicate that perhaps a greater emphasis on auditory perceptual training could be a worthwhile inclusion in the curriculum.

Several of the tests of perceptual efficiency were Piagetian based. The Test of Visual Representation incorporated five euclidean and three topological shapes. The scores on the topological section clustered towards the ceiling with twenty-three subjects gaining the maximum nine points and eighty-one children earning seven or more points. Thus the results on this test support Piaget's contention that topological properties come earlier than do the euclidean properties in a child's understanding of spatial relations.

Neither visual nor tactual representation correlated significantly with mathematics achievement, yet the other two tactual tests of discrimination and memory, each involving geometric shapes did so. The explanation may rest in the lack of motor skills needed for representing shapes by drawing. Such skills were not required for the tests of tactual discrimination and memory.

However another difference exists between these two pairs of tests. The choices in the tests of tactual discrimination and memory were fixed and limited to four. On the other hand an infinite variety of responses and therefore of errors was possible on the two tests of representation. That the mathematics test was a multi-choice one may have been an important commonality in the correlations with tactual discrimination and memory. Furthermore this particular test of mathematics achievement did not feature spatial concepts. The few geometric items included were directed at knowledge of terms and

definitions. This does not, nevertheless, adequately explain the significant correlations between mathematics achievement and one pair of spatially oriented tests and not the other.

The results of the estimation test indicate a general incompetence in such skills, possibly explained by the lack of exposure to estimation activities of a physical nature. The ability to estimate physical quantities is not emphasized in mathematics textbooks, and therefore the tasks in this test were probably outside the children's experience. The inclusion of simpler items may have allowed some of the subjects to demonstrate estimation ability at a lower level of competence. The ability to estimate correlated significantly with visual discrimination and tactual discrimination. The ability to discriminate may be a contributory skill in the ability to estimate.

That conservation, perceptual efficiency and estimation are abilities related to mathematics achievement was proposed and subsequently supported in the review of literature. The results of this study add further weight to this claim. Mathematics achievement correlated significantly with eight of the twelve other variables categorized under conservation, perceptual efficiency and estimation. Conservation of interior volume was one of these. Although the correlation between mathematics achievement and conservation of area only approached significance ($r = .18$, $p < .08$), there was a significant correlation with the Transformed Triangles Test.

The highest correlations were with auditory memory and auditory closure, with auditory discrimination not far behind. The

prominence of the auditory abilities is considered a major finding and will be discussed later in this chapter. Discrimination was the only visual ability which correlated significantly with mathematics achievement. It has already been suggested that visual representation as well as tactual representation was not included in the competencies being measured in the particular mathematics achievement test. Visual memory as tested in this study did not correlate significantly with any other variable and could be dismissed as being irrelevant except for its association with auditory discrimination in the factor analysis.

The other variables which correlated significantly with mathematics achievement were tactual discrimination and memory, and estimation. Thus the choice of abilities related to mathematics achievement seems justified.

Major Findings

The major findings which emerge from this study are:

i. the prominence of the auditory abilities, and

ii. evidence of the qualitative as well as the quantitative differences amongst the children in the sample.

i. The relationship of the auditory abilities to mathematics achievement is seen in a number of the analyses of the data collected for this study. The correlations between mathematics achievement and auditory discrimination, memory and closure were .36, .60, and .60 respectively.

In the stepwise regression analysis in which mathematics

achievement was the criterion variable, auditory closure and auditory memory, along with intelligence, were the main predictors.

The three auditory abilities stood out in the analysis of variance. Auditory memory distinguished the three mathematics achievement groups. Auditory closure showed a difference between the middle and high mathematics groups, while auditory discrimination showed a difference between the middle and low mathematics groups. These differences continued to exist even when differences in intelligence were accounted for.

Although a cause-effect relationship should not be extrapolated from these results, the significance of the relationship between the three auditory abilities and mathematics achievement is pronounced. Educators as far apart as Saunders (1931) and Eagan (1970) have pointed out that the auditory modality is the preferred medium for classroom instruction, and this might provide the explanation for the strong relationship between the auditory abilities and mathematics achievement. Most mathematics teaching would appear to depend largely on the verbal mode. Therefore it seems reasonable to postulate that improvement in auditory abilities may affect advantageously a child's performance in mathematics, on the grounds that he would have a better chance of learning what the teacher is attempting to teach. Those children who are so inefficient at processing auditory input face a serious barrier in classrooms where they are bombarded by sounds which do not convey the intended meaning, if the results obtained on the auditory tests, especially that of closure, are at all typical of daily

performance. The apparent importance of the auditory abilities suggests that a deliberate attempt should be made to provide appropriate experiences which emphasize the auditory modality.

As well as a planned program to improve auditory efficiency, teachers might increase their use of the visual and tactual modalities in their mathematics teaching, in order to reduce the problems of those children low in auditory abilities. The use of materials that children can see and touch may provide not only appropriate learning situations to match a child's strengths, but also to establish or reinforce the meaning of the auditory stimuli coming from the teacher.

ii. The second major finding is concerned with the differences among the children in this study. These differences can be categorized as quantitative or qualitative. For all tests except those measuring conservation and estimation the quantitative differences can be seen in two ways, first, the range of scores compared to the standard deviation, and secondly, by looking at the means for the three mathematics groups on each test.

Visual discrimination provides one example of the wide range of performance by this sample of ninety Grade 3 children. The scores ranged from three to fifty-six, with a mean of 28.7 and a standard deviation of 8.8. A similar spread occurred for auditory discrimination where the number of errors varied from one to twenty-three. The mean errors was 7.2, with a standard deviation of 4.0.

The possible range of scores on the conservation and estimation tests is narrow and does not permit a wide range of performance.

However, where a wide range is possible, as in the examples provided above, the performance of the sample indicates the likely problem of the classroom teacher who is attempting to cater to the individuality of his pupils who often number twenty-five or thirty.

Further evidence of the differences within the sample can be found in the performance of the three mathematics groups. On each test the high mathematics group achieved a higher mean than the low mathematics group. On conservation of interior volume, four of the perceptual efficiencies and estimation, these differences were significant.

The problem of the classroom teacher faced with the range of differences indicated above is further compounded by the qualitative differences which appear among the children in this study. On the nine tests of perceptual efficiency, there were forty-eight children described as top performers on one or more of the tests. Forty-two children were classed as bottom performers. In each case the children were spread over the three mathematics groups, with seven of the top performers only belonging to the low mathematics group and seven of the bottom performers only being high mathematics achievers. Sixteen of the children in the study were categorized as both top and bottom performers. Whereas the tendency of the mean scores to be higher for the high mathematics group and lower for the low mathematics group might justify a simple division of classes into three "homogeneous" groups, the categorization of individual performances as top and bottom indicates that a much more complex grouping of the pupils

in a class may be desirable and even necessary, if the learning of the individual child is the goal of the teacher.

Of the ninety subjects, eighty-four performed better than the mean of the sample on at least two of the abilities tested under conservation, perceptual efficiencies and estimation. The finding that twelve low mathematics achievers were top performers on at least one of the perceptual tests is another heartening fact for the educator. All of these children earned their success in spatially oriented tests, eight of which were of the tactual modality. The presence of these strengths may encourage teachers to consider alternative instructional strategies as being worthwhile to seek. In particular, an emphasis on the tactual modality and spatially oriented tasks may be a useful starting point.

III. SUGGESTIONS FOR TEACHERS

a. The provision of appropriate experiences to increase a child's auditory ability in closure, memory and discrimination is recommended. The teacher needs to identify those children who are at a disadvantage in the classroom because of auditory disabilities. Standardized tests of auditory discrimination and listening comprehension can be added to a teacher's personal repertoire of informal tests. Listening games are one important means of amelioration. In one such game, participating in teams or individually, children can provide elements of sound that make up a word for others to decipher. Sound blending in the early grades usually concentrates on single

phonemes as the elements. It is suggested that various combinations of sounds making up a word be utilized. Those pupils needing remediation will have the experience of integrating the sounds provided by other pupils, as well as the opportunity to analyse a word into a variety of sound units.

b. The design of instructional strategies to capitalize on the tactual strengths that may be found in some low mathematics achievers is encouraged. Some children who have trouble understanding abstract notions such as place value may be more successful if they regrouped easily bundled objects into groups of ten and one hundred, and later worked with structured materials such as Dienes' Multibase Arithmetic Blocks. By recording what they have actually done themselves, some children are more likely to gain insight into our number system.

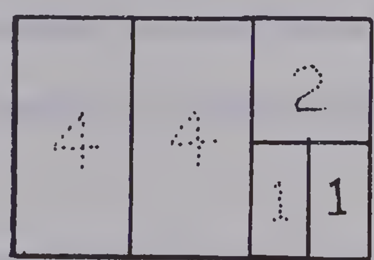
Knowledge of geometric figures can be developed through activities in which children cut out shapes and regroup them, sort prepared shapes according to their properties such as the size of corners, the number of sides or square corners, and make three-dimensional figures from clay, polystyrene foam, drinking straws or cardboard.

c. Instructional strategies in the visual modality to help offset the lack of auditory abilities would call for the use of visual demonstration to reinforce verbal description. The viewing of projectuals on an overhead projector may give insight into some mathematical relationships. The use of the same transparencies by the

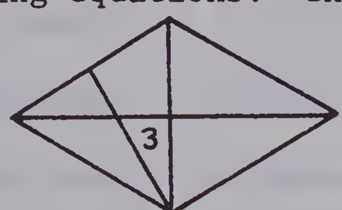
individual child on his desk would allow him to study the numerical information or to manipulate the geometric figures in his own way.

Materials suitable for individual or small group viewing may be preferred to objects which the teacher presents to the whole class. Manipulation of the objects would also seem desirable if the strengths in the tactual modality are to be capitalized upon.

d. The inclusion of spatially oriented tasks in a child's mathematics program would provide another source of mathematical experiences. Geometric figures might be used to teach not only spatial concepts, but also relationships within the number system. For example, numbers can be attached to regions that have a simple size relation. In the illustration below, numbers can be assigned to portions of the space as indicated, and then children can be asked to assign appropriate numbers to other portions in the figures.



A child may want to record the relationships between the numerical values by writing equations. The example below



may generate equations such as

$2 \times 3 = 6$ $\quad$ $9 \div 3 = 3$ $\quad$ $(2 \times 3) + (2 \times 6) = 18$

$9 - 3 = 6$ $\quad$ $\frac{1}{2} \times 6 = 3$ $\quad$ $18 - (6 + 6) = 2 \times 3$

Reusable acetate sheets are economical for these types of activities, because different starting numbers can be selected.

Equilateral triangles, squares and rhomboids that combine to make similar shapes assist the discovery of patterns in the number system, such as odd and even numbers, square and other figurate numbers and remainders.

IV. RECOMMENDATIONS FOR FURTHER RESEARCH

The criterion for this study was the sample's mathematics achievement scores obtained from the Canadian Test of Basic Skills. Many of the items required mechanical-type responses or knowledge of terminology. Reading comprehension was also an important factor. One recommendation for further research is a similar study based on a strong measure of broad mathematical abilities and understandings. This would provide evidence bearing more closely on Piaget's contention of the relationship of conservation to mathematical understanding.

A second recommendation for further research is an extension of the multivariate approach to include conservation of number, in order to establish the status of a child's knowledge of invariance more precisely. Furthermore, a similar multivariate study of children in Grades 2 and 1 might provide evidence on which a mathematics program designed to prevent failure could be based.

This study has shown the strong relationship between the auditory abilities and mathematics achievement. A third recommendation

for further research is a training study in which the cause-effect relationship is investigated. With mathematics achievement as the criterion measure, the experimental group would be provided with extensive experiences planned to improve the efficiency of information processing through the auditory modality.

A fourth recommendation concerns estimation. A rigorous study of children's performances on estimation tasks involving both physical and abstract quantities could serve as the justification of changes in mathematics curricula.

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APPENDIX A

THE RAW MEANS AND ADJUSTED MEANS FOR
THE NINE TESTS OF PERCEPTUAL EFFICIENCY
WHEN INTELLIGENCE IS CONTROLLED

TABLE XIX

RAW AND ADJUSTED MEANS OF THE PERCEPTUAL EFFICIENCY TESTS WITH DIFFERENCES IN INTELLIGENCE CONTROLLED FOR THE HIGH, MIDDLE AND LOW MATHEMATICS GROUPS (N=90)

		Mathematics Groups					
		High		Middle		Low	
		Raw	Adj.	Raw	Adj.	Raw	Adj.
VISUAL	Discrimination	33.57	31.23	28.60	28.22	23.90	26.61
	Memory	25.07	25.02	24.47	24.46	23.83	23.88
	Representation	26.80	25.19	27.97	27.70	26.37	28.23
AUDITORY	Discrimination	5.97	6.01	6.30	6.31	9.30	9.25
	Memory	48.97	47.44	44.13	43.89	38.20	39.97
	Closure	14.20	13.49	9.53	9.41	7.80	8.62
TACTUAL	Discrimination	5.40	5.14	4.63	4.59	4.53	4.83
	Memory	5.77	5.57	5.00	4.97	4.77	4.99
	Representation	28.20	26.70	26.03	25.99	27.33	29.07

APPENDIX B

CORRELATION MATRIX FOR THE MAJOR VARIABLES, FOR THE HIGH, MIDDLE, AND LOW MATHEMATICS ACHIEVEMENT GROUPS, AND THE TOTAL SAMPLE

TABLE XX

CORRELATION MATRIX FOR THE MAJOR VARIABLES

			Conservation		Visual			Auditory			Tactual			
		Mathematics Achievement	Area	Interior Volume	Discrimination	Memory	Representation	Discrimination	Memory	Closure	Discrimination	Memory	Representation	Estimation
Mathematics Achievement	H		.02	.04	.05	.05	.37*	.40*	.27	.29	.17	.05	.53**	.09
	M		.23	.12	.11	.09	.09	.03	.12	.14	.28	.01	.23	.19
	L		.25	.35	.22	.13	.05	.06	.27	.26	.04	.43*	.27	.26
	T		.18	.30**	.44**	.17	.05	.36**	.60**	.60**	.23*	.28*	.16	.39**
Conservation of Area	H			.73**	.20	.03	.17	.01	.22	.08	.03	.20	.06	.15
	M			.68**	.40	.02	.02	.007	.06	.05	.02	.06	.18	.14
	L			.73**	.29	.19	.26	.16	.25	.25	.16	.01	.12	.30
	T			.73**	.04	.10	.18	.12	.21*	.19	.002	.007	.12	.14
Conservation of Interior Volume	H				.27	.21	.45*	.28	.37*	.16	.10	.17	.13	.08
	M				.33	.27	.11	.003	.19	.20	.22	.31	.03	.08
	L				.26	.04	.20	.19	.30	.05	.20	.28	.01	.46**
	T				.004	.08	.26**	.23*	.29**	.26**	.12	.18	.06	.23*
VISUAL Discrimination	H					.33	.01	.23	.28	.20	.03	.05	.20	.22
	M					.14	.07	.01	.41*	.29	.04	.14	.05	.10
	L					.08	.03	.17	.19	.16	.05	.09	.20	.39*
	T					.13	.04	.26**	.48**	.42**	.13	.10	.15	.29**
VISUAL Memory	H						.08	.32	.11	.02	.11	.07	.003	.02
	M						.13	.31	.13	.14	.30	.55**	.14	.02
	L						.10	.18	.07	.23	.03	.28	.02	.19
	T						.05	.04	.01	.19	.11	.16	.08	.10
VISUAL Representation	H							.51**	.33	.15	.20	.03	.47**	.09
	M							.08	.07	.01	.24	.25	.37*	.005
	L							.03	.36*	.01	.24	.01	.06	.25
	T							.04	.24*	.05	.02	.04	.25*	.12
AUDITORY Discrimination	H								.46**	.28	.28	.18	.50**	.01
	M								.24	.04	.12	.21	.05	.11
	L								.31	.08	.04	.18	.08	.10
	T								.42**	.26**	.17	.23*	.10	.17
AUDITORY Memory	H									.41*	.06	.09	.48**	.05
	M									.42*	.04	.11	.08	.08
	L									.17	.06	.42*	.30	.44*
	T									.55**	.11	.26**	.25*	.35**
AUDITORY Closure	H										.16	.05	.62**	.41*
	M										.28	.04	.01	.49**
	L										.21	.07	.06	.07
	T										.31**	.16	.23*	.25*
TACTUAL Discrimination	H											.21	.11	.10
	M											.15	.02	.30
	L											.42*	.27	.08
	T											.30**	.12	.24*
TACTUAL Memory	H												.15	.07
	M												.25	.25
	T												.34	.26
	L												.26**	.13
TACTUAL Representation	H													.07
	M													.29
	L													.50**
	T													.04
Estimation	H													
	M													
	L													
	T													

**Significant at the .01 level
*Significant at the .05 level

APPENDIX C

CORRELATION MATRIX BETWEEN THE VARIABLES INCLUDED IN THE FACTOR ANALYSIS

TABLE XXI

CORRELATION MATRIX BETWEEN THE VARIABLES INCLUDED IN THE FACTOR ANALYSIS

	1	2	3	4	5	6	7	8	9
1	1.000	0.721	0.451	0.169	-0.370	0.612	0.612	0.233	0.282
2	0.721	1.000	0.480	0.125	-0.260	0.536	0.495	0.235	0.239
3	0.451	0.480	1.000	0.128	-0.267	0.485	0.427	0.130	0.106
4	0.169	0.125	0.128	1.000	0.037	0.006	0.189	0.112	0.164
5	-0.370	-0.260	-0.267	0.037	1.000	-0.426	-0.262	-0.170	-0.235
6	0.612	0.536	0.485	0.006	-0.426	1.000	0.557	0.107	0.260
7	0.612	0.495	0.427	0.189	-0.262	0.557	1.000	0.311	0.235
8	0.233	0.235	0.130	0.112	-0.170	0.107	0.311	1.000	0.308
9	0.282	0.239	0.106	0.164	-0.235	0.260	0.161	0.308	1.000
10	0.121	0.205	0.149	0.080	-0.102	0.250	0.235	0.121	0.264
11	0.082	0.136	-0.072	0.023	-0.074	0.150	0.107	-0.070	-0.105
12	0.259	0.288	-0.008	0.160	-0.157	0.239	0.242	0.073	-0.117
13	0.261	0.262	0.014	0.054	-0.207	0.280	0.251	0.117	0.149
14	0.328	0.256	-0.022	0.104	-0.255	0.294	0.256	0.116	0.198
15	0.394	0.297	0.297	0.102	-0.176	0.363	0.253	0.241	0.131
16	0.709	0.806	0.497	0.079	-0.355	0.623	0.557	0.239	0.190
17	0.098	0.304	0.041	0.048	-0.104	0.244	0.050	0.023	0.042

	10	11	12	13	14	15	16	17
1	0.121	0.082	0.259	0.261	0.328	0.394	0.709	0.098
2	0.205	0.136	0.288	0.262	0.256	0.297	0.806	0.304
3	0.149	-0.072	-0.008	0.014	-0.022	0.297	0.497	0.041
4	0.080	0.023	0.160	0.054	0.104	0.102	0.079	0.048
5	-0.102	-0.074	-0.157	-0.207	-0.255	-0.176	-0.355	-0.104
6	0.250	0.150	0.239	0.280	0.294	0.363	0.623	0.244
7	0.235	0.107	0.242	0.251	0.256	0.253	0.557	0.050
8	0.121	-0.070	0.073	0.117	0.116	0.241	0.239	0.023
9	0.264	-0.105	0.102	0.149	0.198	0.131	0.190	0.042
10	1.000	-0.108	-0.117	0.084	0.038	0.042	0.238	0.247
11	-0.108	1.000	0.611	0.492	0.555	0.100	0.064	0.165
12	-0.117	0.611	1.000	0.774	0.787	0.164	0.213	0.157
13	0.084	0.492	0.774	1.000	0.893	0.212	0.283	0.267
14	0.038	0.555	0.787	0.893	1.000	0.245	0.277	0.243
15	0.042	0.100	0.164	0.212	0.245	1.000	0.303	0.125
16	0.238	0.064	0.213	0.283	0.277	0.303	1.000	0.160
17	0.247	0.165	0.157	0.267	0.243	0.125	0.160	1.000

TABLE XXI (continued)

Variables

1. Mathematics Achievement
2. Intelligence
3. Visual Discrimination
4. Visual Memory
5. Auditory Discrimination
6. Auditory Memory
7. Auditory Closure
8. Tactual Discrimination
9. Tactual Memory
10. Tactual Representation
11. Barns Test - Conservation of Area
12. Transformed Trianges Test - Conservation of Area
13. Rearranged Cube Test - Conservation of Interior Volume
14. Islands Test - Conservation of Interior Volume
15. Estimation
16. Reading
17. Visual Representation

A P P E N D I X   D

RAW DATA FOR THE TOTAL SAMPLE

TABLE XXII

RAW DATA FOR THE TOTAL SAMPLE

HIGH GROUP
(N=30)

						Conservation									Visual Representation									
	Sex	Age (months)	Math Achievement	I.Q.	Reading	Barns	Triangles	Area	Cube	Islands	Interior Volume	Total	Visual Discrimination	Visual Memory	Euclidean	Topological	Total	Auditory Discrimination Errors	Auditory Memory	Auditory Closure	Tactual Discrimination	Tactual Memory	Tactual Representation	Estimation
Angelina	G	109	4.55	116	4.35	N	N	N	N	N	N	N	38	23	17	6	23	07	42	12	6	4	20	9
Brian	B	103	4.65	112	4.35	C	C	C	C	C	C	C	33	26	20	7	27	08	51	14	6	3	25	8
Claire	G	105	5.35	120	5.75	C	C	C	C	C	C	C	31	23	18	8	26	02	56	16	6	7	29	6
Dayle	B	108	4.55	109	3.80	N	C	P	P	P	P	P	43	28	21	7	28	05	48	11	8	6	21	8
Dermot	B	109	4.65	119	5.85	C	C	C	C	C	C	C	29	21	24	7	31	03	56	05	5	6	23	10
Elaine	G	104	4.45	121	5.00	N	N	N	N	N	N	N	33	22	15	5	20	09	38	10	6	3	25	9
Fausto	B	103	4.55	108	4.65	N	P	P	C	C	C	P	31	23	19	7	26	04	48	16	4	6	33	8
Gary	B	108	4.55	121	5.75	C	N	P	N	P	P	P	35	29	19	8	27	04	46	14	6	7	30	8
Gilda	G	102	4.80	115	4.35	C	C	C	C	C	C	C	32	22	18	8	26	04	55	18	5	3	26	5
Gina	G	102	4.90	114	5.65	N	N	N	N	N	N	N	56	28	20	9	29	03	51	16	5	6	31	8
Gladys	G	103	5.55	122	4.95	N	C	P	P	C	P	P	35	26	24	6	30	01	58	17	6	7	38	9
Gregory	B	099	4.90	119	4.25	N	P	N	N	N	N	P	31	26	18	9	27	11	50	11	0	3	28	6
Hanna	G	101	5.15	117	5.30	C	N	P	C	C	C	P	31	23	22	9	31	06	54	17	6	4	33	7
Jack	B	110	5.10	117	5.00	C	N	N	N	N	N	P	33	23	22	6	28	05	42	12	6	6	28	7
Joanne	G	107	4.95	121	5.35	C	C	C	C	C	C	C	35	28	24	9	33	07	49	20	5	5	31	7
Kelly	B	109	4.60	102	3.40	C	C	C	C	C	C	C	38	26	19	6	25	05	55	10	5	8	27	8
Lisa	G	109	4.55	102	4.80	N	N	N	N	N	N	N	34	25	16	6	22	03	56	15	4	7	31	6
Mahke	B	107	4.55	123	6.55	C	C	C	C	C	C	C	32	25	21	7	28	07	53	19	7	7	31	5
Marc	B	102	5.30	109	2.40	N	N	N	N	N	N	N	31	25	21	7	28	04	44	15	6	7	35	6
Maria	G	104	4.60	112	4.35	N	N	N	N	N	N	N	37	30	19	5	24	10	47	11	6	7	27	7
Mark	B	107	4.30	106	4.50	C	P	P	C	C	C	P	28	28	19	8	27	12	50	10	2	5	24	7
Michael	B	101	4.35	113	3.75	P	P	P	C	C	C	P	37	19	22	8	30	01	55	18	5	6	31	4
Natalia	G	103	4.60	125	5.10	C	C	C	C	C	C	C	27	23	21	8	29	08	43	14	7	8	30	8
Paddy	B	109	4.40	119	4.15	C	C	C	N	N	N	P	32	29	21	8	29	15	43	16	4	6	21	4
Rikardo	B	104	4.55	116	4.75	N	N	N	N	N	N	N	35	20	18	7	25	07	53	18	6	8	28	5
Roman	B	104	4.35	118	3.75	N	P	P	N	N	N	P	32	27	20	7	27	09	34	06	5	7	20	6
Sharon	G	109	4.55	125	6.15	C	C	C	C	C	C	C	41	28	22	7	29	02	48	18	4	4	31	1
Sherry	G	110	4.90	127	5.15	N	N	N	N	N	N	N	43	28	22	8	30	05	57	20	7	8	35	6
Vinko	B	108	4.90	116	4.15	N	P	P	P	C	P	P	08	23	18	7	25	06	39	13	6	7	22	3
Zenon	B	099	4.80	126	5.85	N	N	N	N	N	N	N	26	25	24	7	31	06	48	14	8	2	32	4

MIDDLE GROUP
(N=30)

						Conservation									Visual Representation									
	Sex	Age (months)	Math Achievement	I.Q.	Reading	Barns	Triangles	Area	Cube	Islands	Interior Volume	Total	Visual Discrimination	Visual Memory	Euclidean	Topological	Total	Auditory Discrimination Errors	Auditory Memory	Auditory Closure	Tactual Discrimination	Tactual Memory	Tactual Representation	Estimation
Andre	B	101	3.65	115	3.25	C	C	C	C	C	C	C	05	29	21	7	28	05	31	04	4	7	31	4
Barbara	G	107	3.65	119	3.55	N	P	P	N	N	N	P	34	25	24	8	32	03	45	06	6	5	33	3
Bernard	B	122	3.80	071	1.85	C	C	C	N	P	P	P	12	29	20	4	24	04	44	08	4	5	31	5
Carmela	G	109	3.80	105	3.35	C	C	C	P	C	P	P	36	24	22	6	28	04	52	15	6	4	30	10
Cheryl	G	110	3.40	097	2.65	N	N	N	N	N	N	N	31	30	20	9	29	03	39	08	5	5	33	5
Cheryll	G	096	3.60	104	4.75	N	C	P	C	C	C	P	24	24	20	8	28	08	53	12	6	4	26	4
Chris	B	107	3.65	102	3.50	C	C	C	C	C	C	C	25	23	20	5	25	09	34	10	5	5	21	5
Clara	G	102	3.80	115	3.35	N	N	N	N	N	N	N	36	25	21	9	30	08	37	11	7	5	15	6
Colleen	G	100	3.55	110	4.40	N	N	N	N	N	N	N	30	22	22	8	30	11	40	10	4	1	28	5
Collette	G	101	3.50	121	4.40	N	N	N	N	N	N	N	41	25	23	3	26	09	53	07	2	6	26	3
Darrell	B	100	3.85	121	4.40	C	P	P	N	N	N	P	20	18	21	9	30	09	48	09	1	4	33	6
Daryl	B	106	3.85	083	2.20	C	N	P	N	C	P	P	27	24	22	6	28	05	44	05	5	5	27	5
Diane	G	109	3.60	099	4.45	N	N	N	N	N	N	N	31	23	17	8	25	05	46	08	6	5	26	6
Evonne	G	103	3.65	107	5.65	C	N	P	N	N	N	P	27	22	22	7	29	05	47	08	2	4	20	7
Geno	B	103	3.35	094	1.65	C	P	P	P	P	P	P	03	24	23	9	32	09	41	08	7	7	28	7
Giselle	G	107	3.65	113	5.45	C	P	P	N	N	N	P	42	19	20	9	29	06	57	12	5	4	32	3
Helga	G	100	3.75	125	5.45	N	C	P	C	C	C	P	27	32	21	9	30	05	50	20	7	7	36	8
Joann	G	114	3.45	093	2.60	C	C	C	C	C	C	C	24	22	21	8	29	05	41	11	3	6	24	3
John	B	111	3.75	095	2.45	N	N	N	N	N	N	N	32	26	19	6	25	08	46	08	4	6	22	3
Lori-Ann	G	105	3.65	114	4.30	N	N	N	N	N	N	N	29	21	22	7	29	04	41	11	4	4	27	8
Michelle	G	106	3.85	114	4.00	C	P	P	N	P	P	P	33	27	24	7	31	08	49	09	5	6	28	4
Mike	B	100	3.55	113	3.20	N	N	N	N	N	N	N	29	18	18	7	25	06	42	03	2	3	22	4
Noreen	G	102	3.50	109	4.00	N	N	N	N	N	N	N	29	29	22	9	31	04	41	10	5	6	28	6
Patrick	B	101	3.55	107	3.20	C	C	C	C	C	C	C	31	27	22	8	30	07	36	06	2	7	30	2
Perry	B	110	3.80	109	3.80	C	C	C	P	P	P	P	29	21	18	7	25	09	38	09	7	2	18	8
Rickie	B	103	3.65	107	3.00	C	N	P	N	N	N	P	27	22	22	7	29	05	47	08	2	4	20	7
Rosanna	G	105	3.70	110	3.90	N	N	N	N	N	N	N	35	23	21	8	29	05	40	07	7	7	30	3
Sheila	G	100	3.75	115	4.60	C	C	C	C	C	C	C	37	27	20	9	29	05	47	13	7	6	29	7
Teresa	G	110	3.50	098	4.10	N	N	N	N	N	N	N	40	23	20	4	24	08	48	18	3	6	26	6
Tommy	B	112	3.55	100	2.50	C	P	P	N	N	N	P	27	24	16	7	23	05	48	10	3	3	30	6

LOW GROUP
(N=30)

	Sex	Age (months)	Math Achieve-ment	I.Q.	Reading	Conservation: Barns	Triangles	Area	Cube	Islands	Interior Volume	Total	Visual Discrimination	Visual Memory	Visual Representation: Euclidean	Topological	Total	Auditory Discrimination Errors	Auditory Memory	Auditory Closure	Tactual Discrimination	Tactual Memory	Tactual Representation	Estimation
Alain	B	111	2.30	084	3.00	N	N	N	C	N	P	P	27	21	22	8	30	06	44	10	5	5	30	6
Angela	G	105	2.45	084	3.05	N	N	N	N	N	N	N	03	24	20	8	28	09	39	05	7	6	29	5
Anna	G	108	2.75	079	2.35	P	N	P	N	N	N	P	19	28	21	7	28	07	32	14	7	4	29	4
Arthur	B	110	2.25	083	2.40	N	N	N	N	N	N	N	05	21	18	9	27	07	35	05	3	4	19	1
Barry	B	099	2.75	099	1.60	C	C	C	C	C	C	C	23	29	23	9	32	18	36	08	5	5	31	8
Charlotte	G	102	2.50	081	2.60	N	N	N	N	N	N	N	18	26	18	6	24	08	39	09	5	6	33	2
Dahryll	B	103	2.15	102	2.60	C	C	C	N	N	N	P	24	27	21	9	30	12	31	14	6	4	20	3
Dale	B	100	2.15	086	2.50	C	P	P	P	N	P	P	25	23	21	7	28	08	39	02	3	5	32	8
Debra	G	116	2.25	063	2.00	N	N	N	N	N	N	N	24	19	19	9	28	05	28	03	5	4	29	3
Donald	B	110	2.85	111	4.65	N	N	N	P	N	P	P	31	22	17	9	26	09	34	07	6	8	31	7
Ezia	G	106	2.30	081	1.55	N	N	N	N	N	N	N	13	22	20	6	26	18	34	05	4	6	27	3
Franco	B	103	2.85	099	2.70	N	N	N	N	N	N	N	18	23	24	9	33	10	44	07	4	5	30	4
Frank	B	102	2.30	106	2.95	N	N	N	N	N	N	N	25	22	19	9	28	08	43	07	6	7	24	3
Gail	G	116	2.20	077	2.15	N	N	N	N	N	N	N	21	25	15	7	22	14	28	05	3	2	15	2
Helen	G	108	2.35	080	1.90	N	N	N	N	N	N	N	28	24	22	8	30	09	38	06	6	5	34	6
Joseph	B	106	1.90	098	2.70	C	N	P	N	N	N	P	20	23	18	9	27	14	36	11	4	1	26	4
Karen	G	110	2.85	101	2.85	C	P	P	N	N	N	P	29	20	21	8	29	06	44	14	5	5	27	3
Linda	G	098	2.60	094	2.85	N	N	N	N	N	N	N	31	22	19	7	26	06	33	09	6	5	27	5
Marque	B	101	2.45	112	2.40	C	C	C	C	C	C	C	34	23	18	6	24	03	46	09	7	7	32	7
Mick	B	109	2.35	078	1.75	C	N	P	N	N	N	P	25	23	22	9	31	13	43	12	4	3	30	6
Natalie	G	111	2.60	084	3.90	C	C	C	C	C	C	C	25	26	22	7	29	07	32	09	3	3	22	4
Richard	B	109	2.85	075	2.00	N	P	P	N	P	P	P	28	26	19	4	23	05	42	14	3	5	28	8
Robert	B	115	2.45	092	2.40	P	N	P	N	N	N	P	31	25	17	7	24	22	40	04	7	5	28	5
Roberto	B	106	2.25	091	2.90	P	N	P	N	N	N	P	30	27	23	5	28	05	31	03	2	1	25	4
Rosemarie	G	101	2.80	084	1.60	N	N	N	N	N	N	N	25	24	18	9	27	23	31	08	2	4	34	3
Shane	B	106	2.85	101	3.30	C	C	C	C	C	C	C	27	27	22	8	30	09	38	06	2	5	23	5
Therese	G	101	2.85	113	5.10	C	C	C	C	C	C	C	26	24	24	7	31	05	59	08	3	6	29	8
Twyla	G	102	2.55	106	2.70	C	N	P	N	N	N	P	32	29	18	9	27	04	46	09	5	5	28	4
Willie	B	102	2.70	091	2.95	C	C	C	C	C	C	C	26	17	18	4	22	05	39	05	4	7	23	3
Yvonne	G	104	2.25	079	2.55	N	N	N	N	N	N	N	24	23	16	7	23	04	42	06	4	5	25	5

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